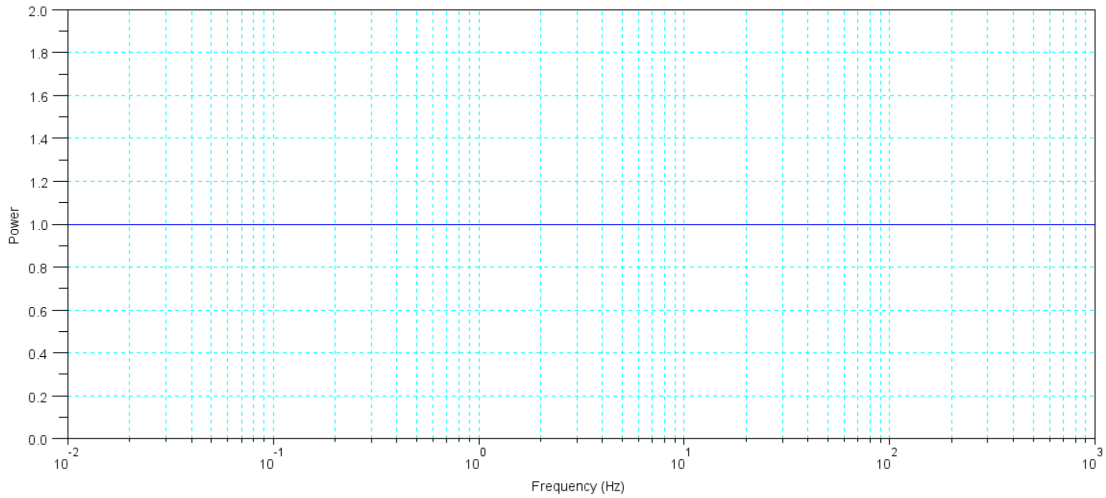
Pink Noise and Noise Cancellation

Problem:

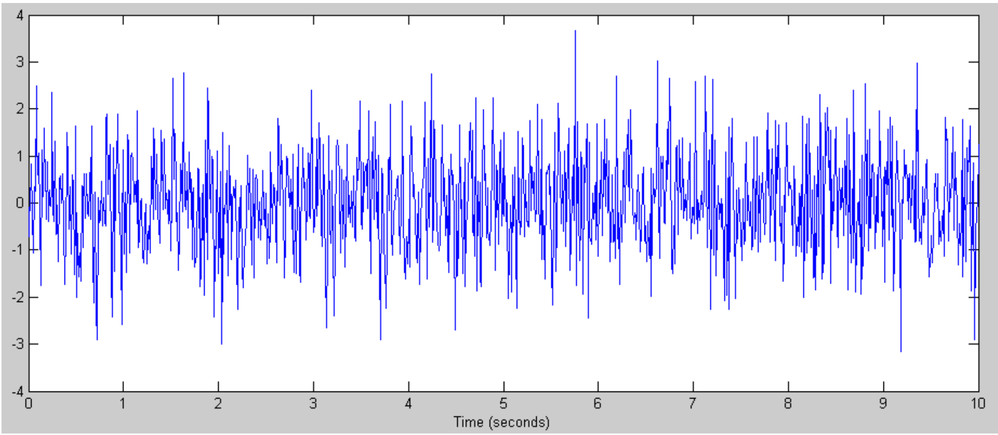
A system has an input disturbance with a distinct spectra. Can you take this information into account?

Pink Noise:

The power spectrum for Gaussian noise is 1:

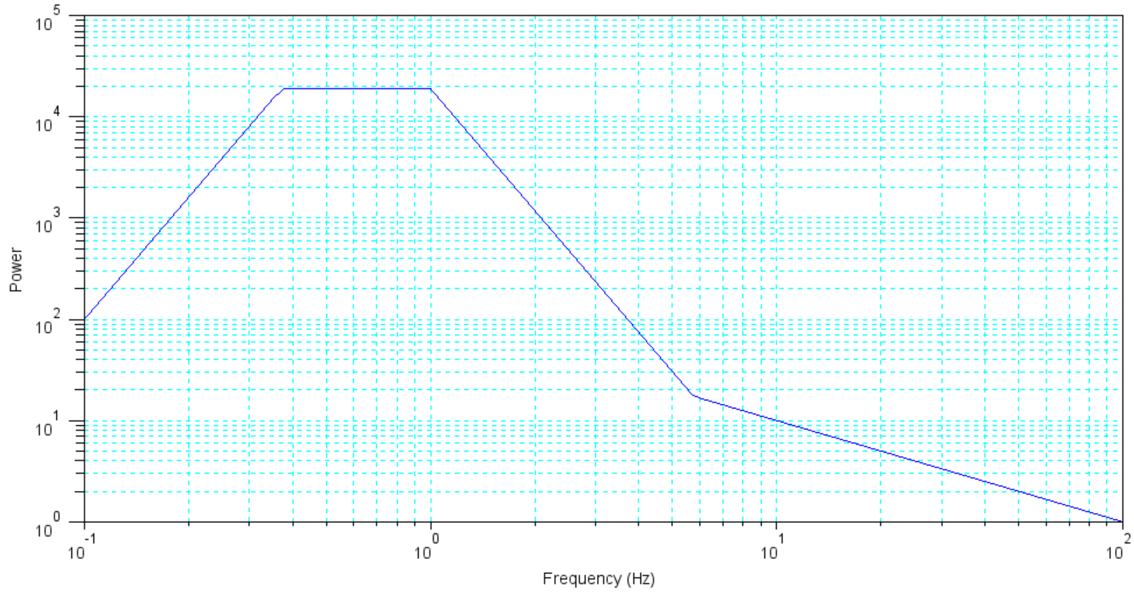


Power Spectrum Density for a disturbance: $d \sim N(0, 1^2)$



Time response for $d \sim N(0, 1^2)$

Some disturbances, such as wind, solar heating, etc. are not uncorrelated white noise like this. Instead, their spectrum has a distinct shape. For example, the Dryden Spectrum is the approximate power of wind buffeting an airplane



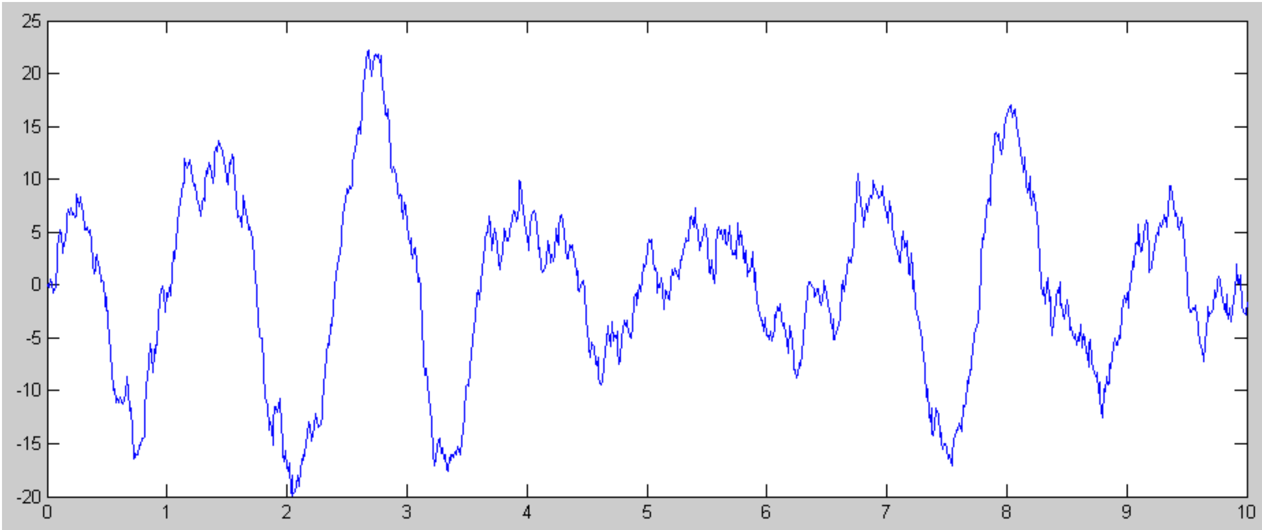
Approximate Power Spectrum of Wind (Dryden Spectrum): *from Control Systems Design* by Bernard Friedland

You can approximate this as white noise passed through a band-pass filter:

$$d \approx \left(\frac{100(s+0.1)}{s^2+10s+25}\right)\eta \qquad\qquad \eta \sim N(0,1)$$

which has the following time response:

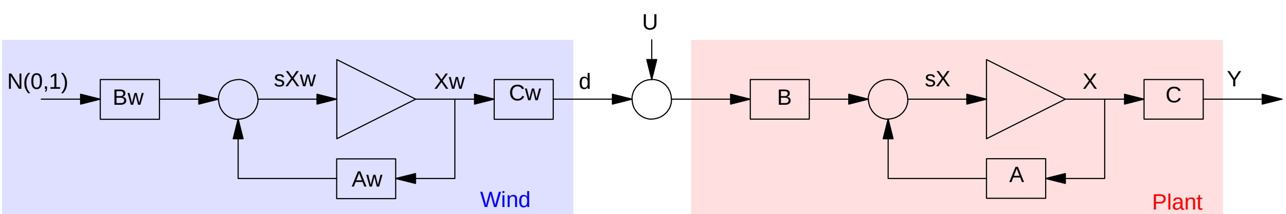
```
>> G = tf([100,10],[1,10,25])

Transfer function:
  1000 s + 10
---------------
s^2 + 10 s + 25

>> y = step3(A,B,C,D,t,X0,N);
>> plot(t,N)
>> plot(t,y)
```



Computer Generated Wind Speed vs. Time

This is termed 'pink noise': noise with a spectral content.

Example: Add pink noise to the 4th-order RC filter:



Plant & Pink Noise Disturbance

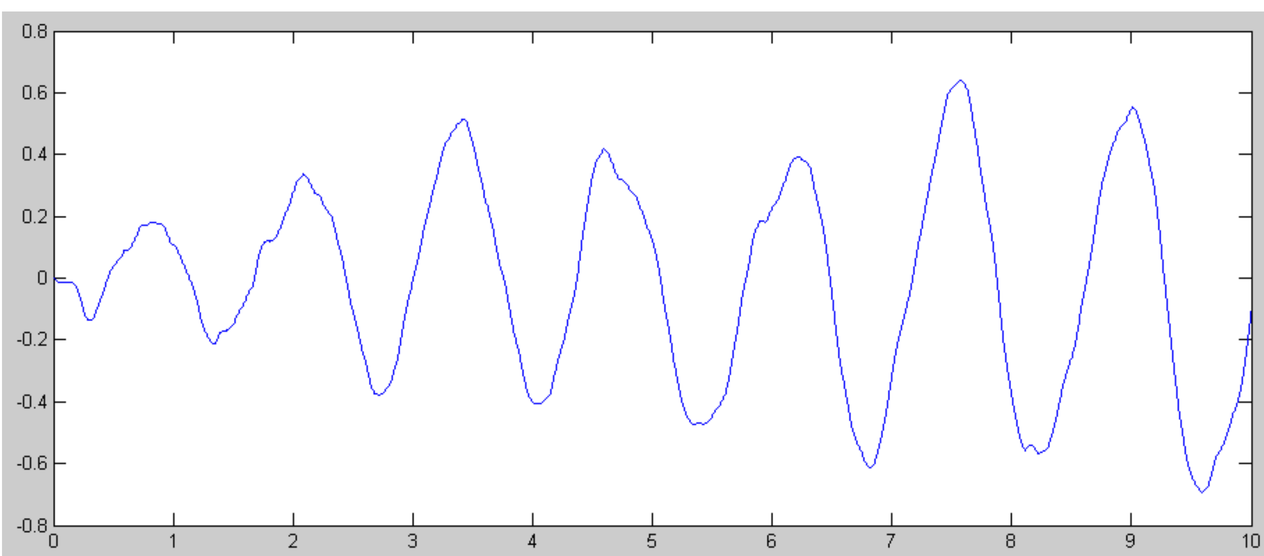
The dynamics of the plant and disturbance with no feedback are:

$$s\begin{bmatrix} X \\ X_w \end{bmatrix} = \begin{bmatrix} A & BC_w \\ 0 & A_w \end{bmatrix}\begin{bmatrix} X \\ X_w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ B_w \end{bmatrix} \eta$$

Example: Gantry System.

Assume the plant is the 4th-order Gantry system with wind applying a force disturbance on the displacement:

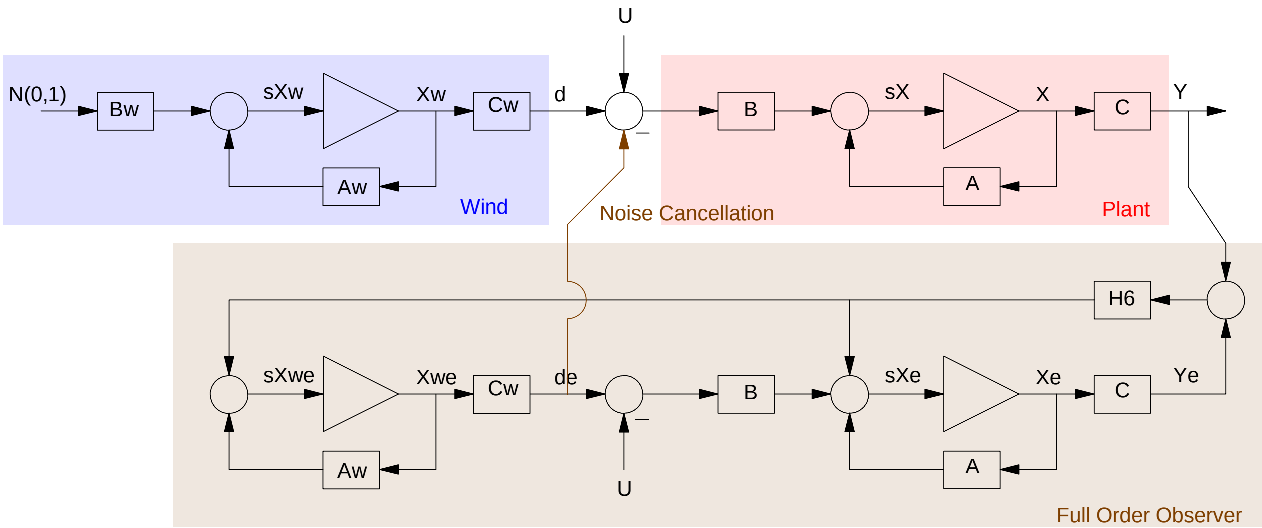
```
W = 1;
V = 0.01;

t = [0:0.001:10]';
N = size(t);

nu = W*randn(N);
ny = V*randn(N);

A = [0,0,1,0;0,0,0,1;0,9.8,0,0;0,-19.6,0,0];
B = [0;0;1;-1];
C = [1,0,0,0];

Aw = [0  1 ; -25  -10];
Bw = [0 ; 100];
Cw = [0.1 1];

A6 = [A, B*Cw ; zeros(2,4), Aw];
B6 = [B ; 0; 0];
C6 = [0,0,1,0,0,0];    % velocity
D6 = 0;

F = [0; 0; 0; 0; Bw];

X0 = zeros(6,1);

y = step3(A6, B6, C6, D6, t, X0, nu);

plot(t,y);
```



Velocity of the Gantry Resulting from Wind Disturbances:

Observer for Noise Cancellation

Now add a full-order observer to estimate the states - including the states of the disturbance:



Plant & Pink Noise (wind) & Full-Order Observer

Plant & Disturbance

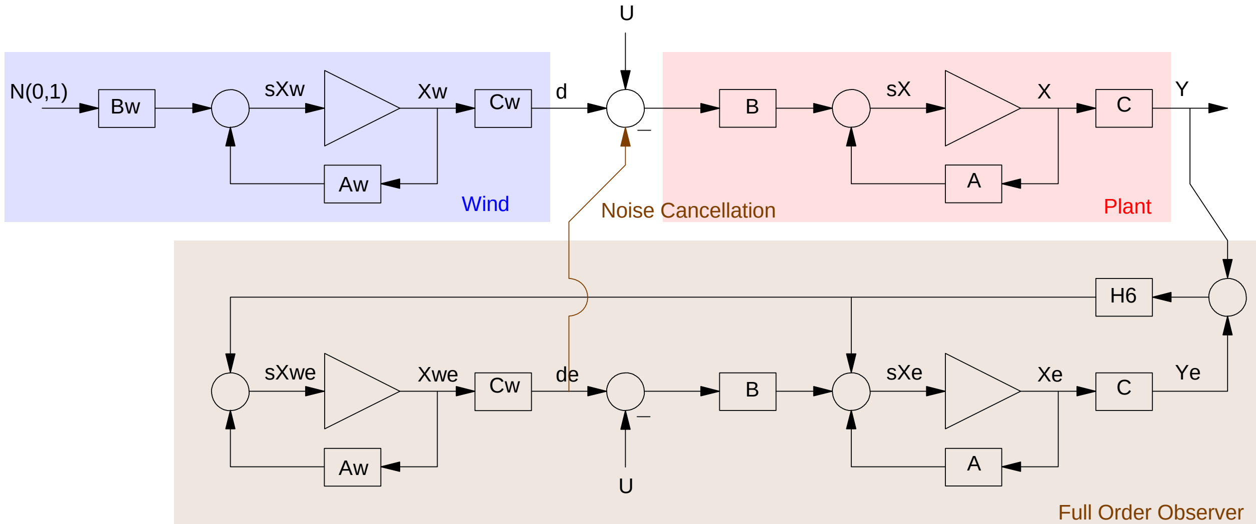
$$\begin{bmatrix} sX \\ sX_w \end{bmatrix} = \begin{bmatrix} A & BC_w \\ 0 & A_w \end{bmatrix} \begin{bmatrix} X \\ X_w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} X \\ X_w \end{bmatrix}$$

Design an optimal observer (Kalman filter)

```
Q = W*F*F'*W;
R = V^2;

H6 = lqr(A6', C6', Q, R)'

      33.7
   -  31.3
     567.2
   - 563.7
    0567.0
    3.432.1
```

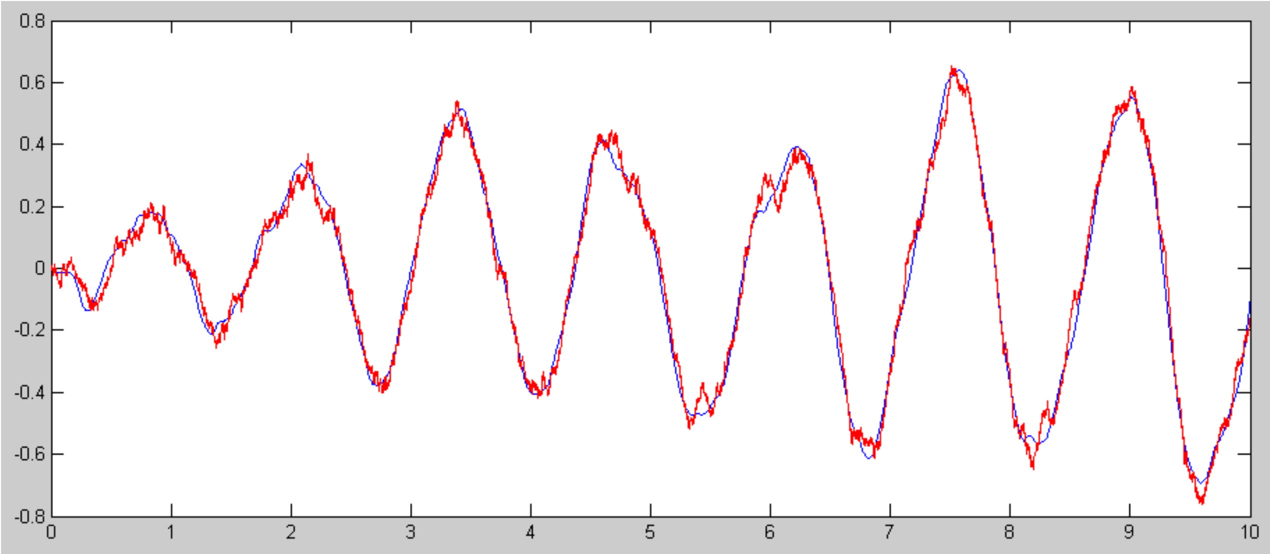
The plant plus the observer is then:



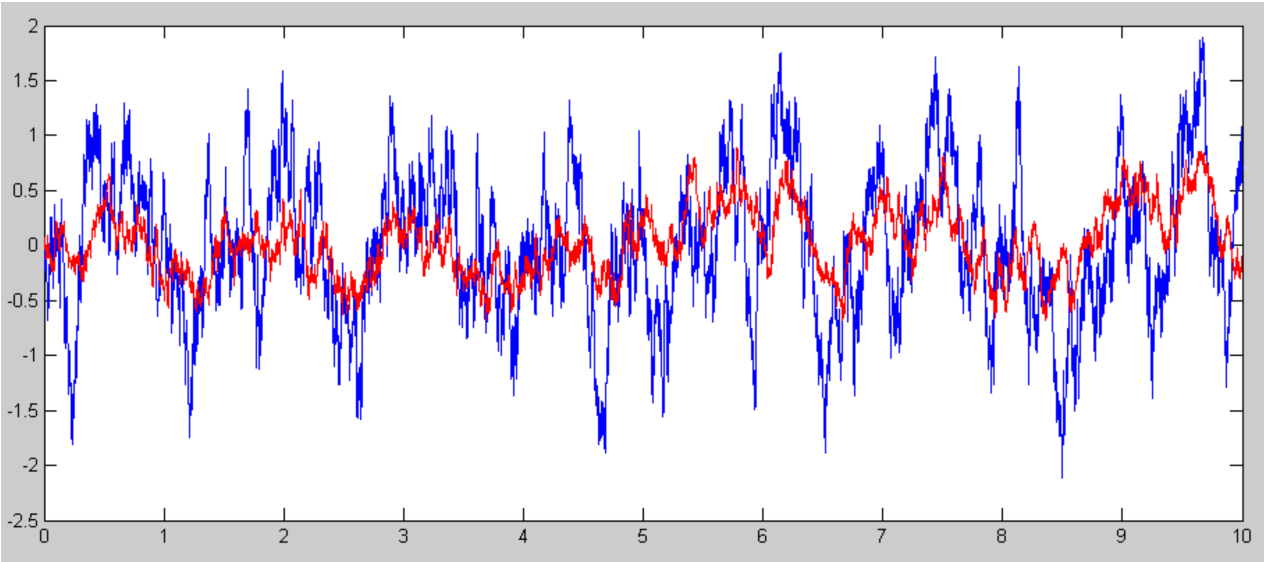
$$\begin{bmatrix} sX \\ sX_w \\ s\hat{X} \\ s\hat{X}_w \end{bmatrix} = \begin{bmatrix} A & BC_w & 0 & 0 \\ 0 & A_w & 0 & 0 \\ H_x C & 0 & A - H_x C & BC_w \\ H_w C & 0 & 0 - H_w C & A_w \end{bmatrix} \begin{bmatrix} X \\ X_w \\ \hat{X} \\ \hat{X}_w \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ B_w \\ 0 \\ 0 \end{bmatrix} \eta$$

Put the plant and observer together:

```
A12 = [A6, zeros(6,6) ; H6*C6, A6-H6*C6];
B12 = [B6, [0;0;0;0;Bw], zeros(6,1) ; B6, zeros(6,1), H6]

C12 = zeros(4,12);
C12(1,3) = 1;     % velocity
C12(2,9) = 1;     % velocity estimate
C12(3,6) = 1;     % d
C12(4,12) = 1;    % d estimate

D12 = zeros(4,3);
X0 = zeros(12,1);
t = [0:0.001:10]';
N = size(t);
U = [zeros(N), nu, ny];

y = step3(A12, B12, C12, D12, t, X0, U);
plot(t,y0(:,1),'b',t,y0(:,2),'r')
```



Cart Velocity (blue) and its estimate (red).



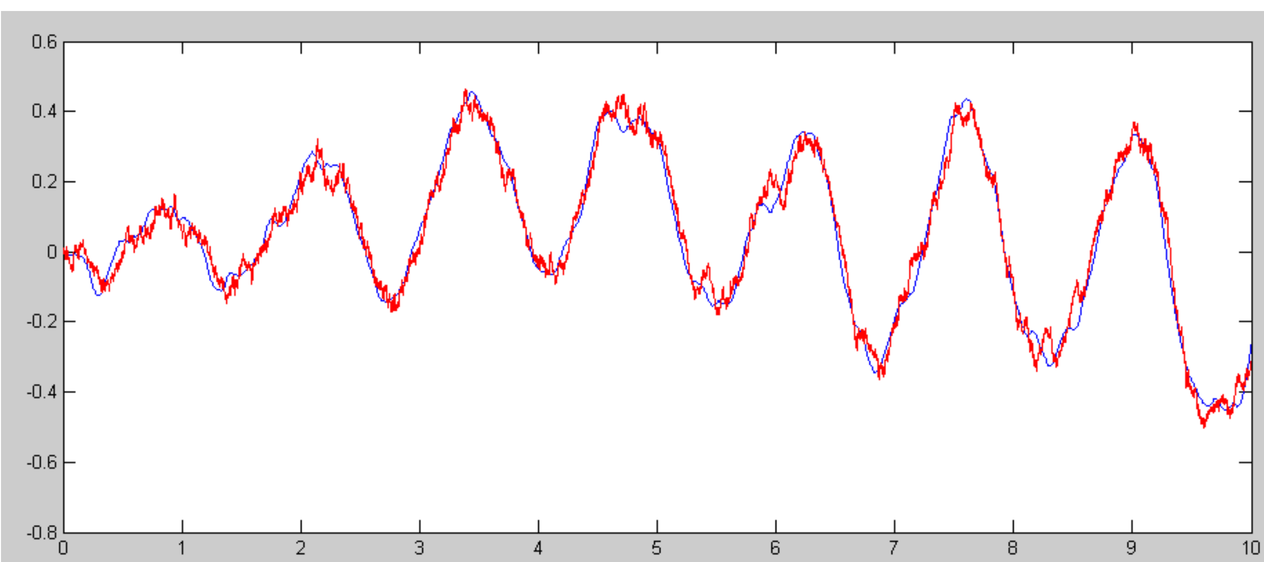
Disturbance (blue) and its estimate (red)

If you know the disturbance, you can cancel it at the input. This results in the system being

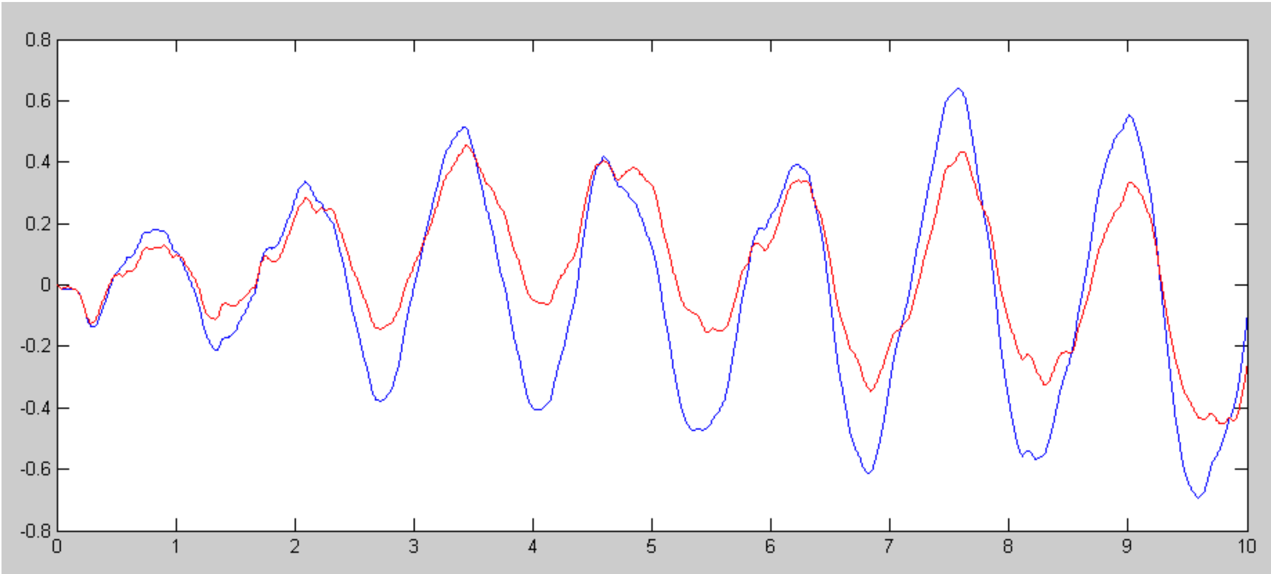
$$\begin{bmatrix} sX \\ sX_w \\ s\hat{X} \\ s\hat{X}_w \end{bmatrix} = \begin{bmatrix} A & BC_w & 0 & \underline{-BC_w} \\ 0 & A_w & 0 & 0 \\ H_xC & 0 & A-H_xC & \underline{0} \\ H_wC & 0 & 0-H_wC & A_w \end{bmatrix} \begin{bmatrix} X \\ X_w \\ \hat{X} \\ \hat{X}_w \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ B_w \\ 0 \\ 0 \end{bmatrix} \eta$$

This should reduce the effect of the disturbance on the output. In Matlab:

```
A12 = [A6, zeros(6,6) ; H6*C6, A6-H6*C6];

A12(1:4,11:12) = -B*Cw;
A12(7:10,11:12) = 0*B*Cw;


B12 = [B6, [0;0;0;0;Bw], zeros(6,1) ; B6, zeros(6,1), H6]
C12 = zeros(4,12);
C12(1,3) = 1;     % dx
C12(2,9) = 1;     % dx estimate
C12(3,6) = 1;     % d
C12(4,12) = 1;    % d estimate

D12 = zeros(4,3);

X0 = zeros(12,1);
t = [0:0.001:10]';
N = size(t);
U = [zeros(N), nu, ny];

y = step3(A12, B12, C12, D12, t, X0, U);
plot(t,y(:,1),'b',t,y(:,2),'r')
```



Plotting the velocity with and without noise cancellation



Velocity due to wind disturbance without noise calcellation (blue) and with noise calcellation (red)