Observers and Disturbance Rejection

NDSU ECE 463/663

Lecture #21

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Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

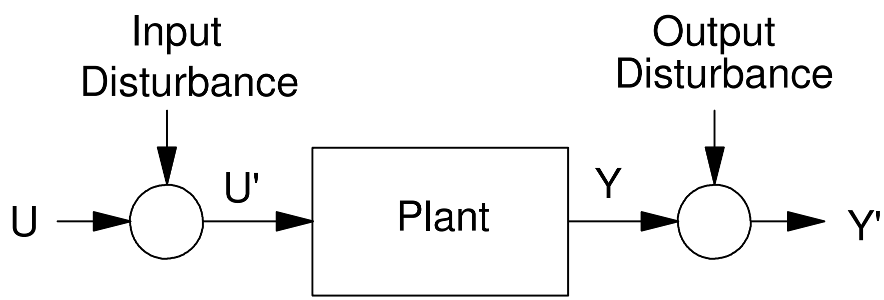
Observers and Disturbances

Suppose a plant has a disturbance

- At the input or
- At the output (sensor).

How do you estimate the states in the presence of such disturbances?

- If you ignore the disturbances, this will throw off the state estimates.
- Bad estimates can adversely affect the feedback control system.



Example: Full-Order observer for a plant with an input disturbance:

Plant

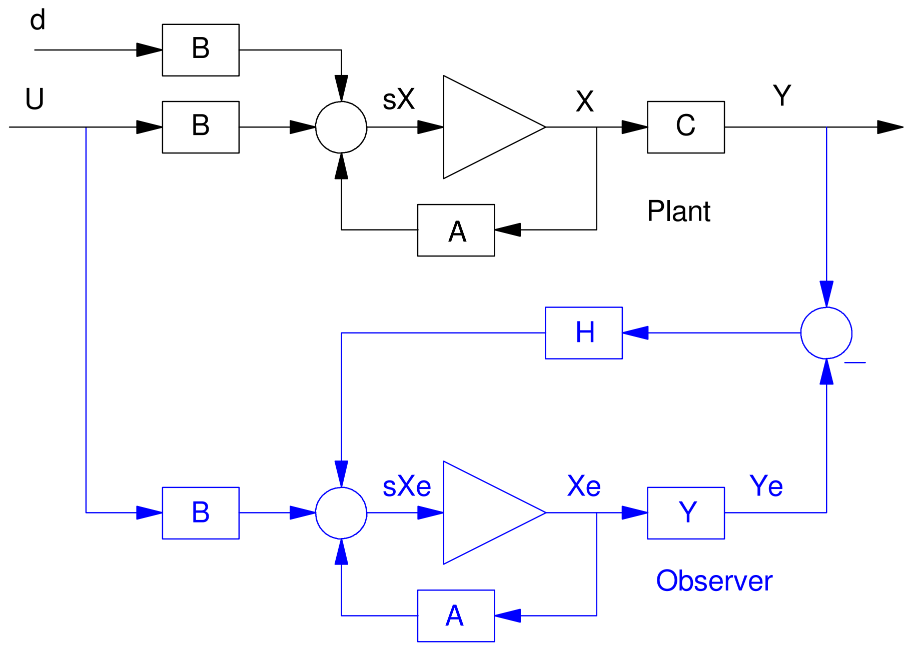
$$sX = AX + B(U + d)$$

$$Y = CX$$

Observer

$$sX_e = AX_e + BU + H(Y - Y_e)$$

$$Y_e = CX_e$$



The error in the state estimate is:

$$E = X - X_e$$

Taking the derivative:

$$sE = sX - sX_e$$

Substituting:

$$sE = (AX + B(U + d)) - (AX_e + BU + H(Y - Y_e))$$

With some algebra:

$$sE = A(X - X_e) + Bd - HC(X - X_e)$$

$$sE = (A - HC)E + Bd$$

Assuming a step input, meaning all the states are constand (sE = 0), then

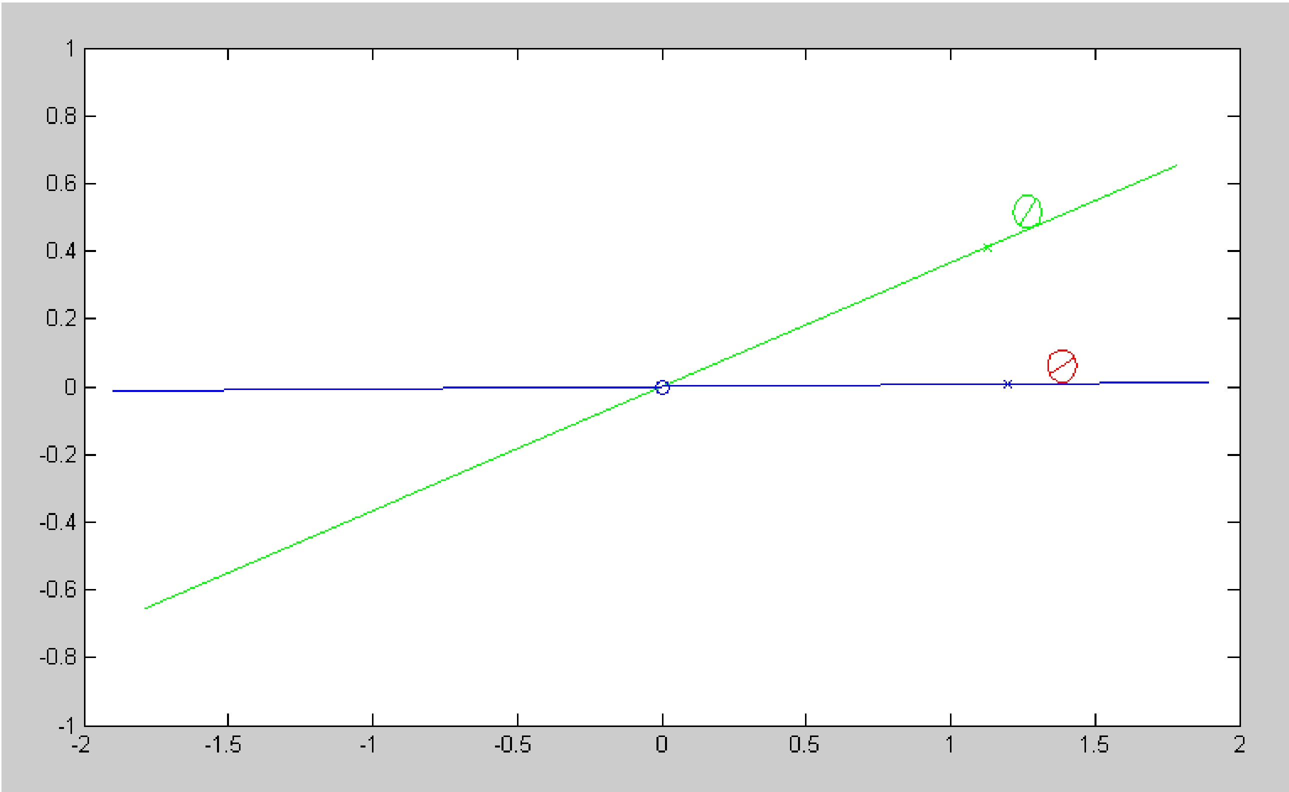
$$E = -(A - HC)^{-1} Bd$$

The disturbance will create an error in the state estimates.

Bad estimates can result in a badly performing feedback system.

Example: Ball and Beam System

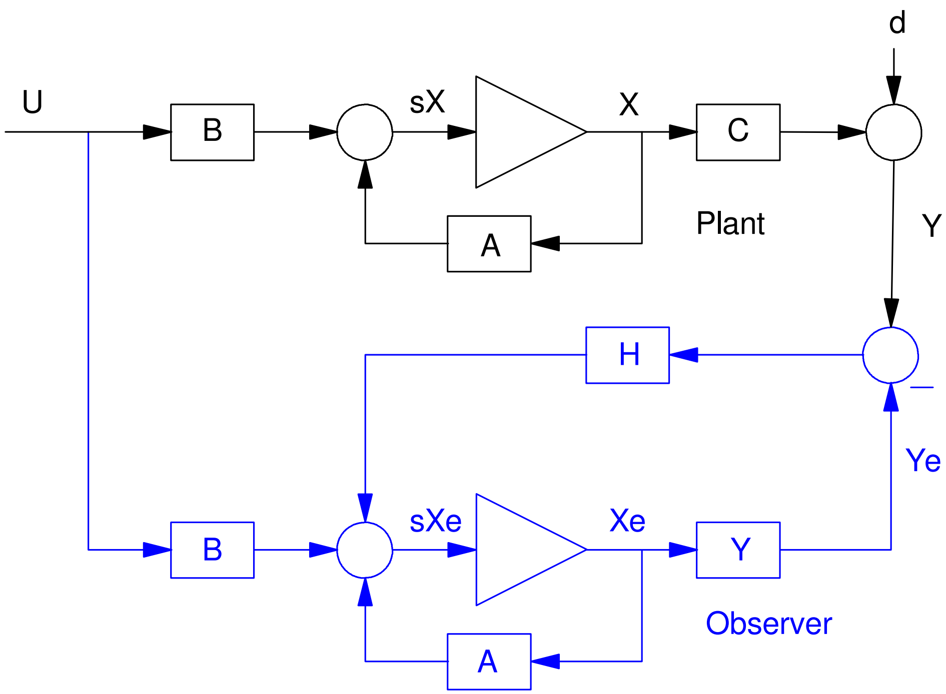
- m = 1.1kg (vs. 1.0kg nominal)
- Green: Observer (where the controller *thinks* the beam is
- Blue: Where the beam *really* is



This is also a problem if you have an output disturbance:

$$sX = AX + BU$$

$$Y = CX + d$$



If the observer ignores this disturbance:

$$sX_e = AX_e + BU + H(Y - Y_e)$$

$$Y_e = CX_e$$

The error is

$$E = X - X_e$$

Taking the derivative:

$$sE = sX - sX_e$$

Substituting:

$$sE = (AX + BU) - (AX_e + BU + H(Y - Y_e))$$

Rewriting this:

$$sE = (AX + BU) - (AX_e + BU + H(CX + d - CX_e))$$

$$sE = (A - HC)E - Hd$$

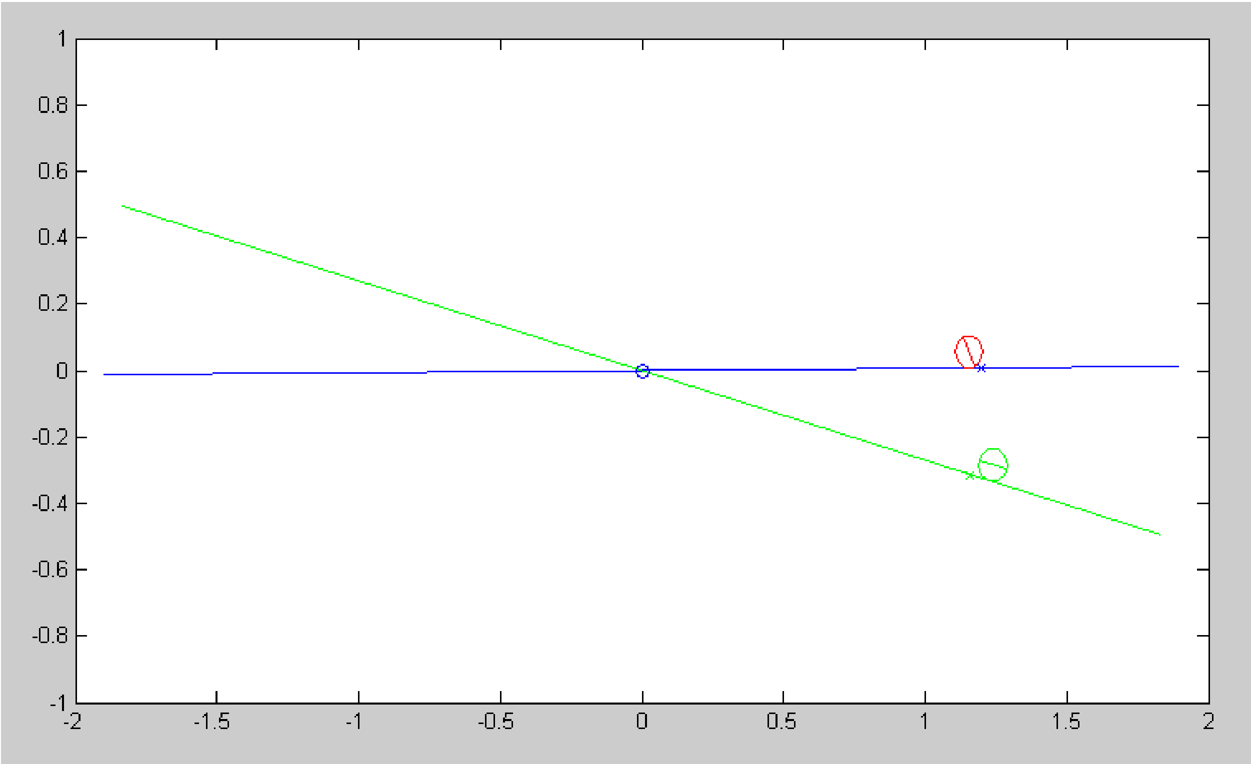
Again, the disturbance affects the error in the state estimates.

- Bad estimates can result in poorly behaving systems.

If a system has a disturbance, it needs to be taken into account with the observer.

Example: Ball and Beam System

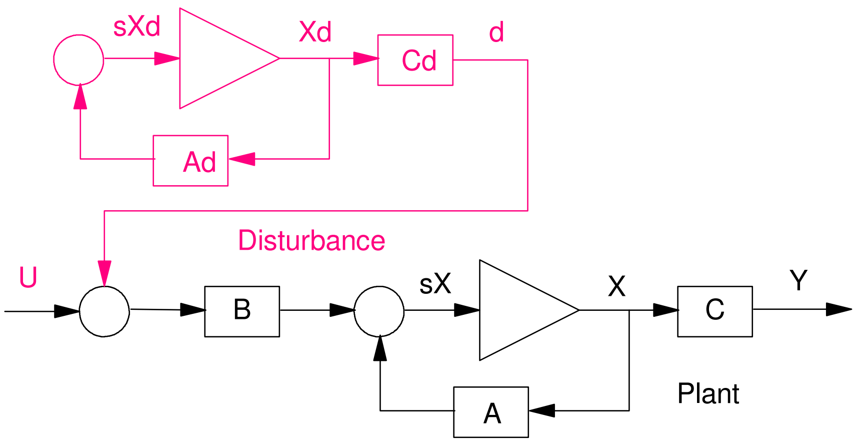
- m = 1.0kg (nominal)
- Y = x + 0.1 *DC offset*
- Blue = Actual State (X)
- Green = State Estimate (Xe)



Case 1: Input Disturbances

Add a dummy state (Xd)

- Initial conditions determine its value
- Ad determines the spectra (constant, sinusoid...)



In state-space, the augmented system is

$$s\begin{bmatrix} X \\ X_d \end{bmatrix} = \begin{bmatrix} A & BC_d \\ 0 & A_d \end{bmatrix}\begin{bmatrix} X \\ X_d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} C & 0 \end{bmatrix}\begin{bmatrix} X \\ X_d \end{bmatrix}$$

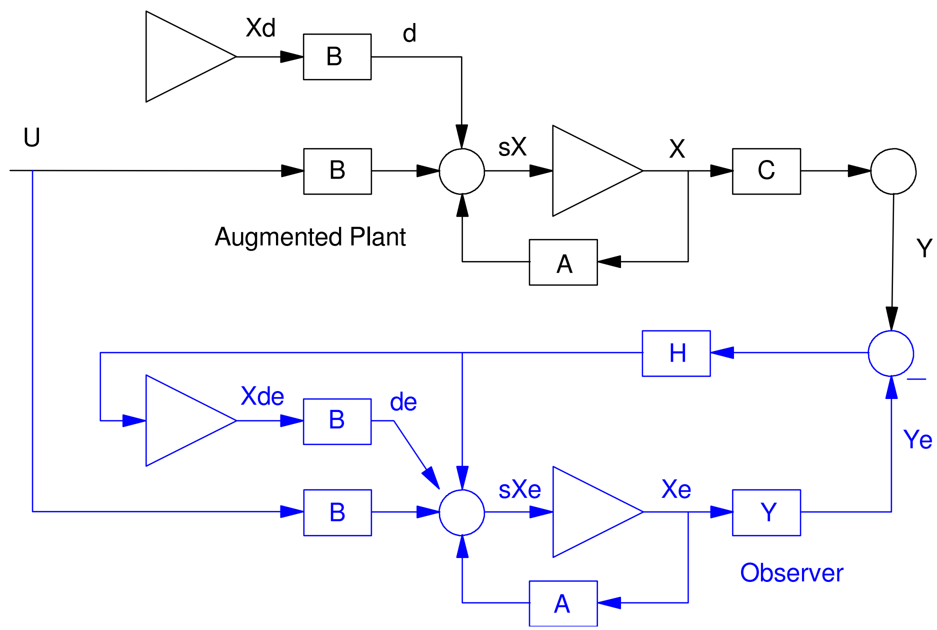
Design a full-order observer for the augmented system

- Plant & Disturbance

Example: 4th-Order heat equation with a constant offset at the input.

First, create the augmented system:

```
A5 = [A,B;0*C,0]

  - 2.     1.     0.     0. :    1.
    1.   - 2.     1.     0. :    0.
    0.     1.   - 2.     1. :    0.
    0.     0.     1.   - 1. :    0.
   - - - - - - - - - - - - - - -
    0.     0.     0.     0. :    0.

B5 = [B; 0]

     1
     0
     0
     0
  - - - -
     0

C5 = [C,0]

    0.     0.     0.     1. :    0.
```



Second, design a full-order observer for the augmented system

- Use Bass-Gura

```
>> H5 = ppl(A5', C5', [-3,-3,-3,-3,-3])'

  147.0000
   72.0000
   27.0000
    8.0000
  243.0000

>> eig(A5-H5*C5)

  -3.0047
  -3.0014 + 0.0044i
  -3.0014 - 0.0044i
  -2.9962 + 0.0027i
  -2.9962 - 0.0027i
```

To simulate for a step input, add a state for U:

$$\begin{bmatrix} sX_5 \\ sX_{5e} \end{bmatrix} = \begin{bmatrix} A_5 & 0 \\ H_5C_5 & A_5 - H_5C_5 \end{bmatrix} \begin{bmatrix} X_5 \\ X_{5e} \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} U$$

Simulate this in Matlab using initial conditions on the disturbance (d = 2) and the input (U = 1). Note that this is now an 10th-order system

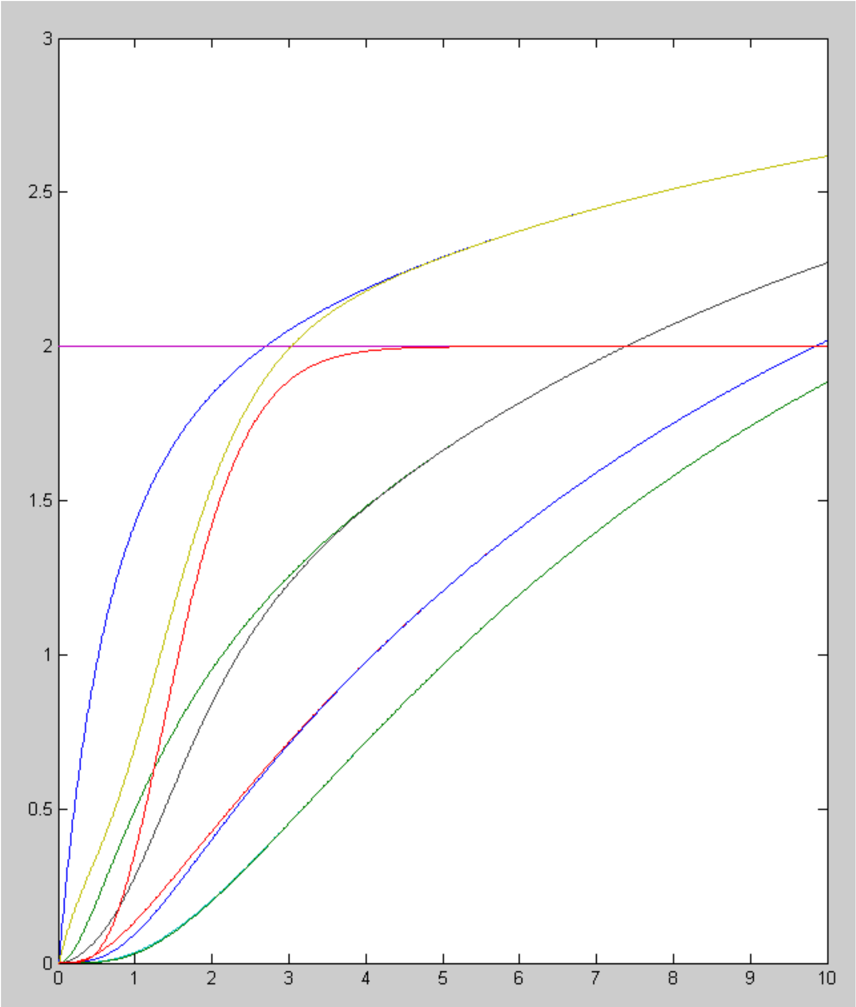
- 4 states for the plant
- 1 for the disturbance
- 5 for the observer (plant plus disturbance)

```
>> A10 = [  A5,   zeros(5,5) ;
           H5*C5, A5-H5*C5 ]

             X       y     d             est(X)        ye   ext(d)
 -2     1     0     0  :   1  :   0     0     0     0  :    0
  1    -2     1     0  :   0  ;   0     0     0     0  :    0
  0     1    -2     1  :   0  ;   0     0     0     0  :    0
  0     0     1    -1  :   0  ;   0     0     0     0  :    0
 - - - - - - - - - - - - - - - - - - - - - - - - - -
  0     0     0     0  :   0  :   0     0     0     0  :    0
- - - - - - - - - - - - - - - - - - - - - - - - - -
  0     0     0   147  :   0  :  -2     1     0  -147  :    1
  0     0     0    72  :   0  :   1    -2     1   -72  :    0
  0     0     0    27  :   0  :   0     1    -2   -26  :    0
  0     0     0     8  :   0  :   0     0     1    -9  :    0
- - - - - - - - - - - - - - - - - - - - - - - - - -
  0     0     0   243  :   0  :   0     0     0  -243  :    0
```

The initial conditions are:

```
X0 = [0;0;0;0;2; 0;0;0;0;0 ]

      0
      0     X(0)  = 0
      0
      0
    - - -
      2     d = 2
    - - -
      0
      0     Xe(0)  = 0
      0
      0
    - - -
      0   estimate of d = 0
```

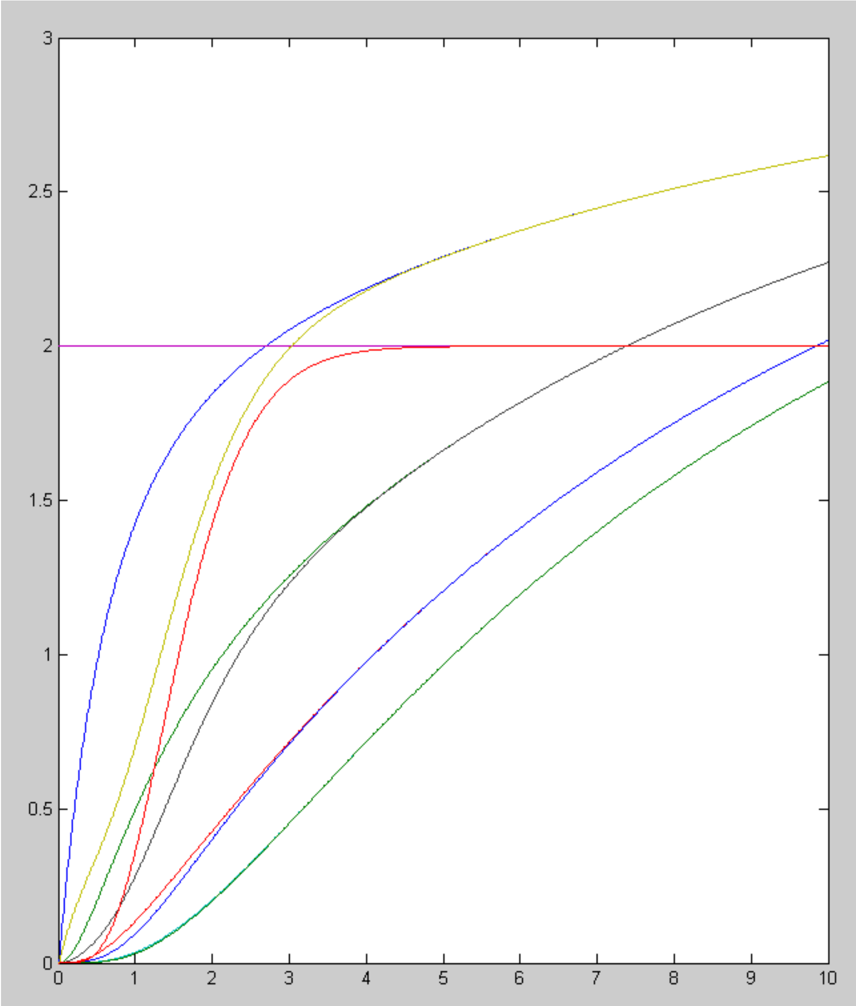
```
B10 = [B ; 0 ; B ; 0];
C10 = eye(10,10);
D10 = zeros(10,1);
U = 0*t + 1;
y = step3(A10,B10,C10,D10,t,X0,U);
plot(t,y);
```



Note that

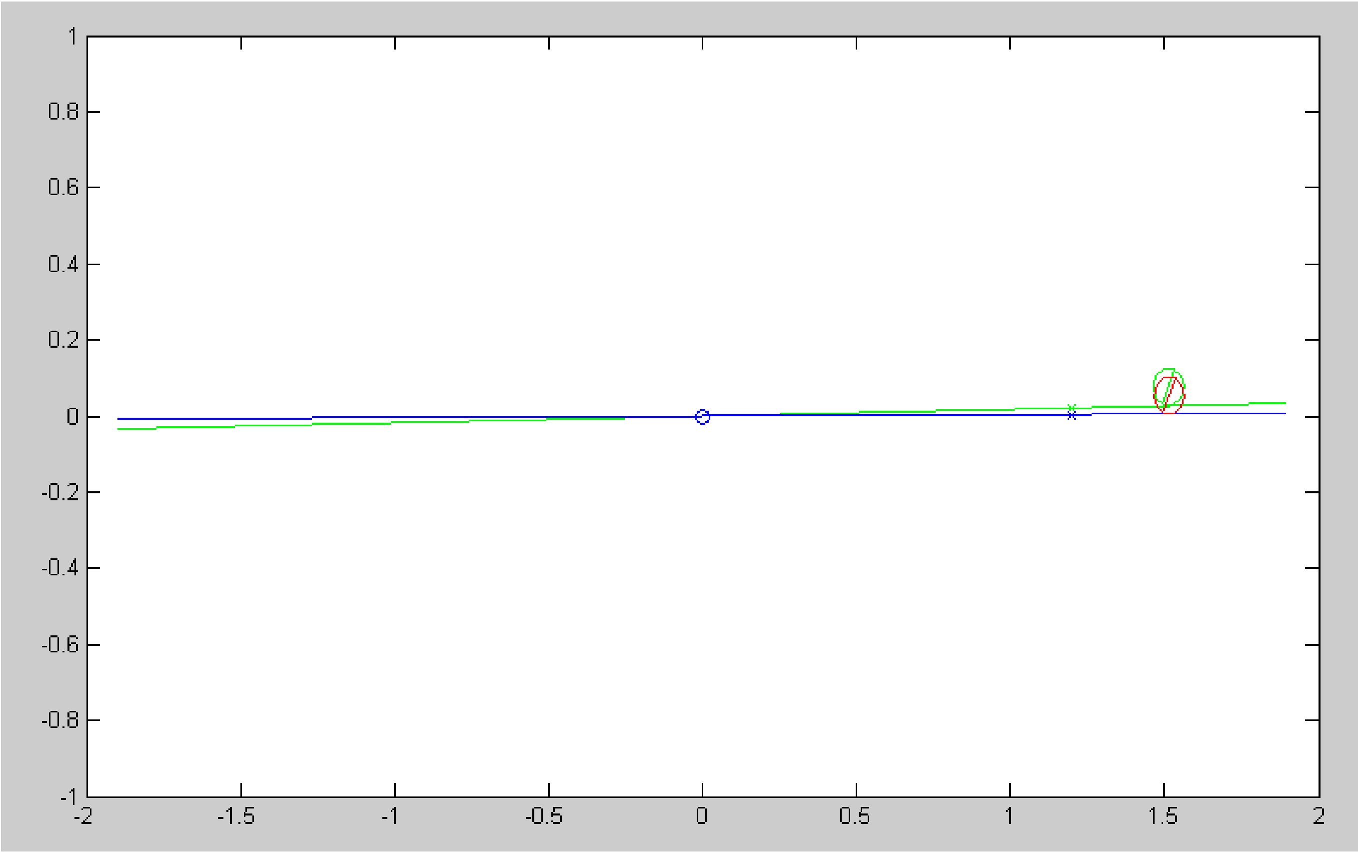
- The disturbance estimates converges in about 4 seconds. This is expected since the observer poles were placed at {-3, -3, -3, -3, -3}
- The disturbance is a constant, 2 (pink line)
- The estimate of the disturbance converges to 2 in about 4 seconds (red line)

All state estimates converge to the actual states in spite of the constant output disturbance.



Ball & Beam Systems

- m = 1.1kg
- Model as a constant input disturbance (torque)
- Observer eventually determines the constant offset (extra mass)



Nonliner Simulation

- Cheat: Use the actual states for the first 5 seconds
- Then switch over to the observer states

```
X = [0.8;0;0;0];
Xe = [X; 0];
dt = 0.01;
t = 0;
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-8.167,0,0,0];
B = [0;0;0;0.8333];
C = [1,0,0,0];
A5 = [A,B;0*C,0];
B5 = [B; 0];
C5 = [C,0];
H = ppl(A5', C5', [-3, -3.2, -3.4, -3.6, -3.8])';
Kx = [-13.9152   42.0017   -8.5718   12.0005];
Kr = -4.1145;
y = [];

while(t < 10)
   Ref = 1.0 + 0.2*sign(sin(t));
   if(t<5)
     U = Kr*Ref - Kx*X;
   else
      U = Kr*Ref - Kx*Xe(1:4);
   end
   dX  = BeamDynamics(X, U);
   dXe = A5*Xe + B5*U + H*(X(1) - Xe(1));
   X = X + dX * dt;
   Xe = Xe + dXe*dt;
   y = [y ; X(1), Xe(1), Ref];
   t = t + dt;
   BeamDisplay20(X, Xe, Ref);
```

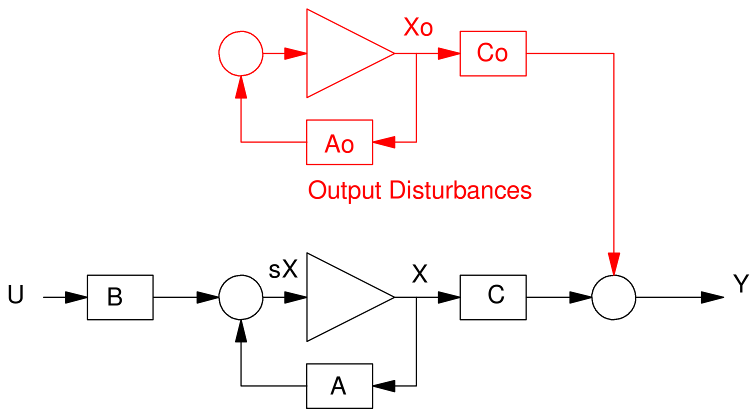
Case 2: Output Disturbance

Next, consider the case where the output (sensor) has a disturbance:

In state-space

$$s\begin{bmatrix} X \\ X_o \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_o \end{bmatrix}\begin{bmatrix} X \\ X_o \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} C & C_o \end{bmatrix}\begin{bmatrix} X \\ X_o \end{bmatrix}$$



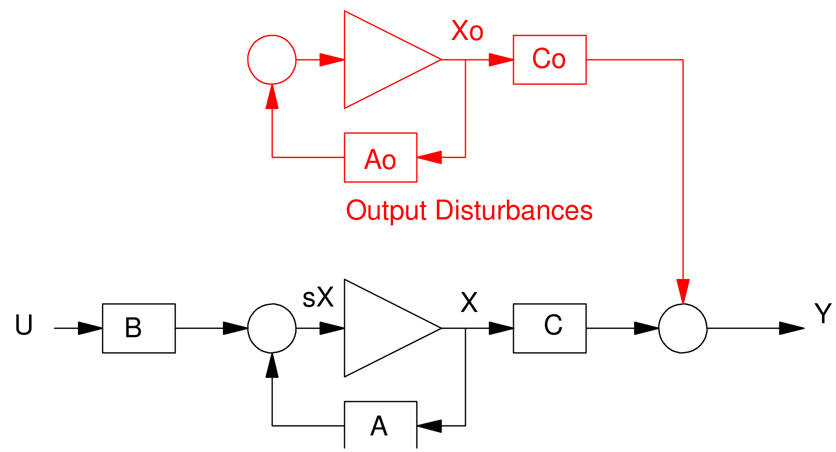
Design a full-order observer the augmented system

Example: Heat equation with

$$Y = x_5 + 2$$

Creating the augmented system

```
A5 = [A,B*0;0*C,0]

   -2     1     0     0  :    0
    1    -2     1     0  :    0
    0     1    -2     1  :    0
    0     0     1    -1  :    0
   - - - - - - - - - - - - -
    0     0     0     0  :    0

B5 = [B ; 0];

    1
    0
    0
    0
   - -
    0

C5 = [0,0,0,1,1]

    0     0     0     1  :    1
```

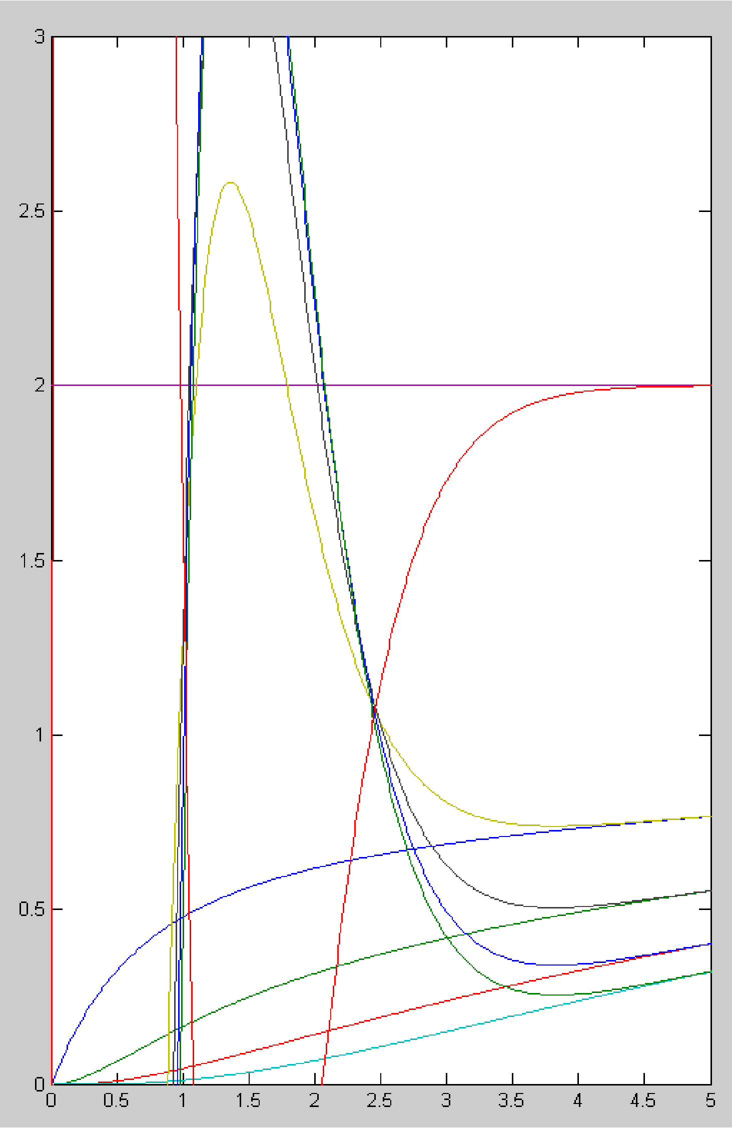


Using Bass-Gura, you can then place the poles at will

```
>> H5 = ppl(A5', C5', [-3, -3, -3, -3, -3])'

H5 =

  -96.0000
 -171.0000
 -216.0000
 -235.0000
  243.0000

>> eig(A5-H5*C5)

ans =

  -2.9863
  -2.9958 + 0.0130i
  -2.9958 - 0.0130i
  -3.0111 + 0.0080i
  -3.0111 - 0.0080i
```

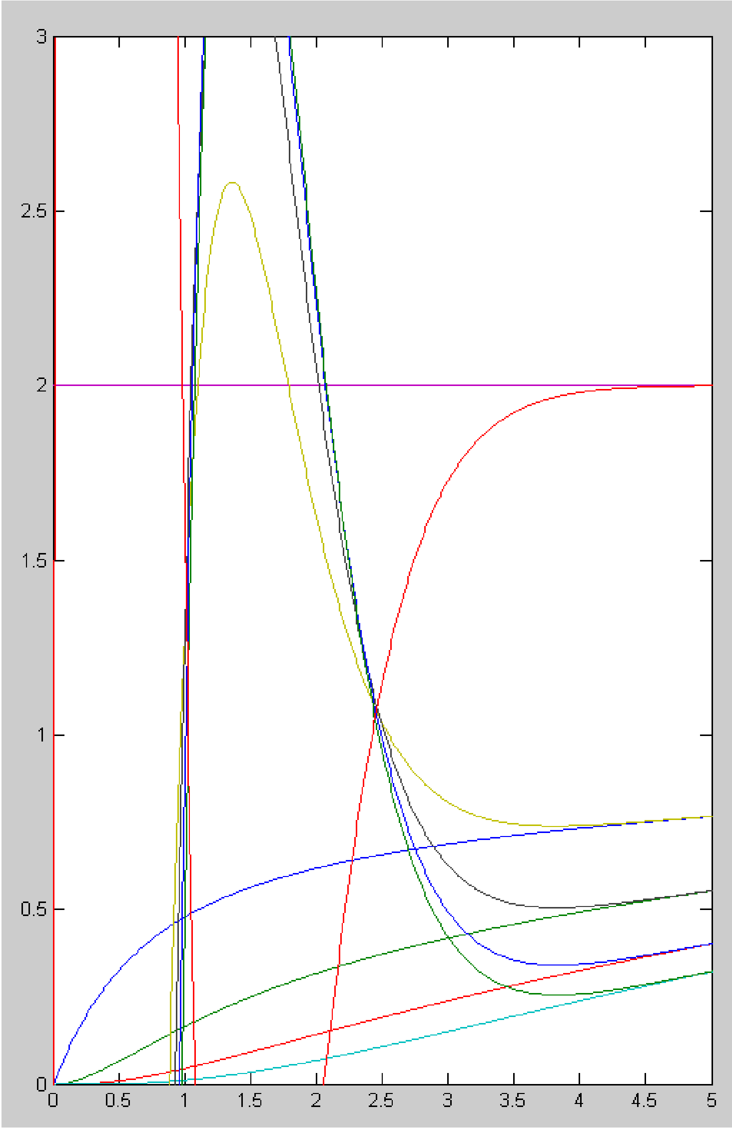
```
>> A10 = [A5, zeros(5,5);     H5*C5, A5-H5*C5]

                X       y       d               ext(X )         ext(d)
 -2     1       0       0 :     0 :     0       0       0       0 :     0
  1    -2       1       0 :     0 :     0       0       0       0 :     0
  0     1      -2       1 :     0 :     0       0       0       0 :     0
  0     0       1      -1 :     0 :     0       0       0       0 :     0
 - - - - - - - - - - - - - - - - - - - - - - - - - - -
  0     0       0       0 :     0 :     0       0       0       0 :     0
 - - - - - - - - - - - - - - - - - - - - - - - - - - -
  0     0       0     -96 :   -96 :    -2       1       0      96 :    96
  0     0       0    -171 :  -171 :     1      -2       1     171 :   171
  0     0       0    -216 :  -216 :     0       1      -2     217 :   216
  0     0       0    -235 :  -235 :     0       0       1     234 :   235
 - - - - - - - - - - - - - - - - - - - - - - - - - - -
  0     0       0     243 :   243 :     0       0       0    -243 :  -243
```

```
>> X0 = [0;0;0;0;2; 0;0;0;0;0]

      0
      0          X(0)
      0
      0
    - - -
      2          d = 2
    - - -
      0
      0          Xe(0)
      0
      0
    - - -
      0          estimate of d

>> B10 = [B5 ; B5];
>> C10 = eye(10,10);
>> D10 = zeros(10,1);
>> U = 0*t + 1;
>> y = step3(A10,B10,C10,D10,t,X0,U);
>> plot(t,y);
```



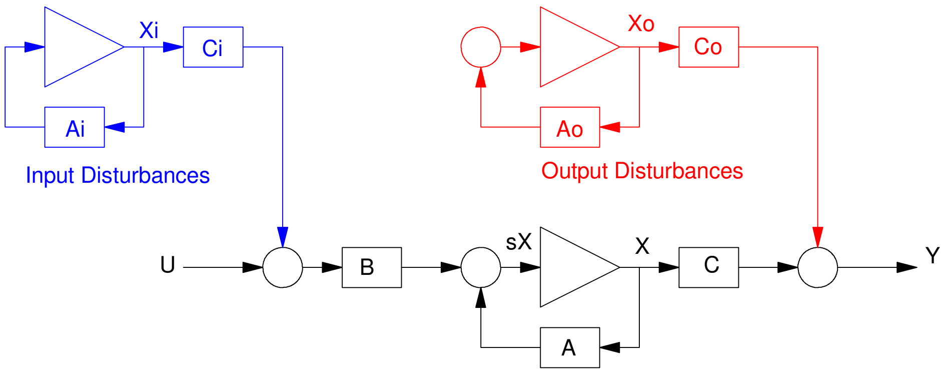
Note that

- The disturbance estimates converges in about 4.3 seconds. This is expected since the observer poles were placed at {-3, -3, -3, -3, -3}
- The disturbance is a constant, 2 (pink line)
- The estimate of the disturbance converges to 2 (red line) in about 4.3 seconds
- All states converge in spite of the constant output disturbance.



Case 3: Input and Output Disturbances

Finally, consider the case where the system has both input and output disturbances:



In state-space:

$$s\begin{bmatrix} X \\ X_i \\ X_o \end{bmatrix} = \begin{bmatrix} A & BC_i & 0 \\ 0 & A_i & 0 \\ 0 & 0 & A_o \end{bmatrix}\begin{bmatrix} X \\ X_i \\ X_o \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} C & 0 & C_o \end{bmatrix}\begin{bmatrix} X \\ X_i \\ X_o \end{bmatrix}$$

Note that there is a problem here.

- Use th PBH test
- The system is observable *only* if the spectra of Ai and Ao are disjoint
- You *cannot* determine a constant disturbance at both the input and output

$$\rho\left(\begin{bmatrix} C \\ A-\lambda I \end{bmatrix}\right) = N$$

$$\rho\left(\begin{bmatrix} A-\lambda I & BC_i & 0 \\ 0 & A_i-\lambda I & 0 \\ 0 & 0 & A_o-\lambda I \\ \cdots & \cdots & \cdots \\ C & 0 & C_o \end{bmatrix}\right) = N$$

Summary

If you have input disturbances,

- Add states to model the disturbance
- Design a full-order observer for the augmented system

If you have output disturbances,

- Add states to model the disturbance
- Design a full-order observer for the augmented system

If you have both input and output disturbances

- You're out of luck - The system isn't observable
- You have to guess how much is input and how much is output