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# **Follow-On Courses**

## **NDSU ECE 463/663**

**Lecture #35**

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Please visit [Bison Academy](#) for corresponding  
lecture notes, homework sets, and solutions

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## What's Next?

Once you have a background in Modern Control, the following courses are open to you:

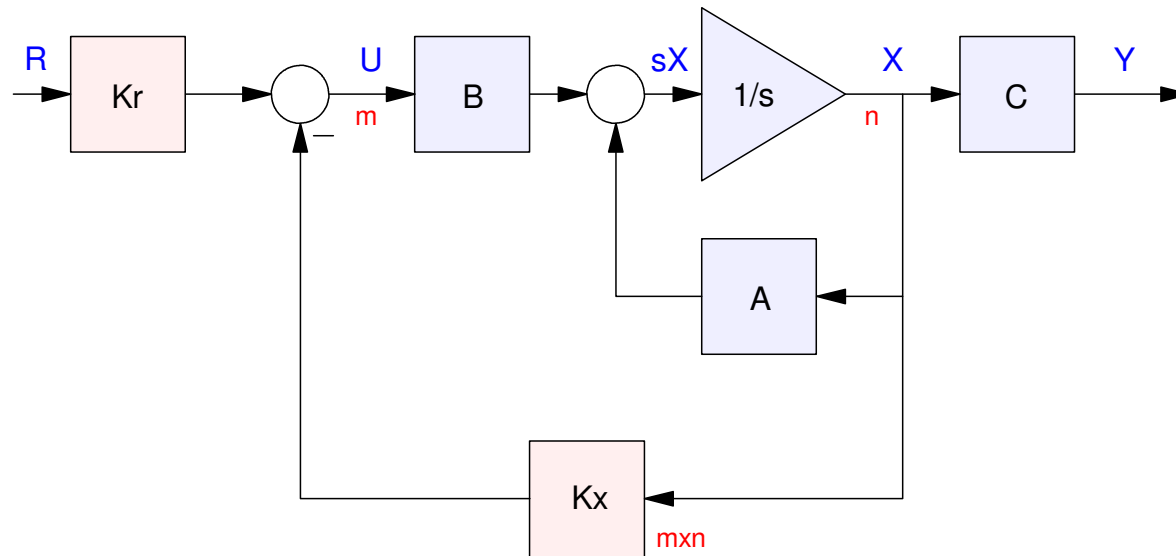
- Multivariable Control
  - Nonlinear Control
  - Robust Control
  - Adaptive Control
  - Self-Tuning Regulators
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# Multivariable Control

Problem: With LQR techniques and multiple inputs, you end up with a full-rank matrix  $K_x$

- Each input requires knowledge of each sensor

$$K_x = \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix}$$



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How to you constrain  $Kx$  so that some terms are zero?

- Only certain sensors are used by each input?

$$K_x = \begin{bmatrix} a & b & 0 & 0 \\ 0 & 0 & c & d \end{bmatrix}$$

Optimal result is the solution to the algebraic ricatti equation

- i.e. it is a solution to the Euler LaGrange equation
  - No closed-form solution exists
  - Have to solve using numerical methods
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# Nonlinear Control

LQR techniques apply to linear systems

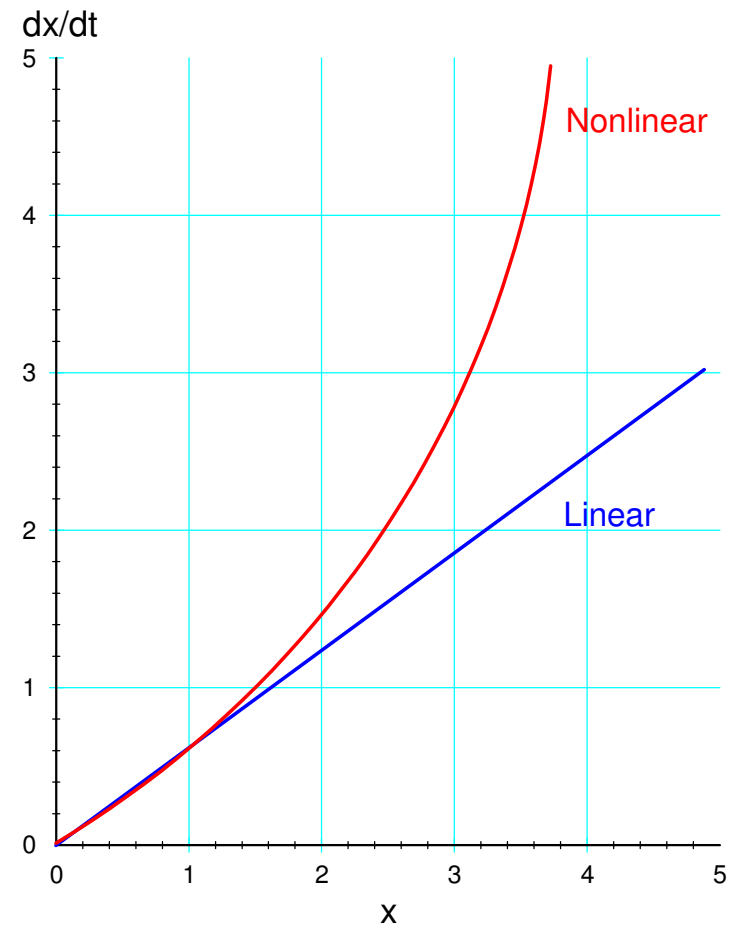
What if the plant is nonlinear?

$$\dot{x} = \sin(x)$$

$$\dot{x} = -x^3$$

How do you guarantee stability for these systems?

How do you find the "optimal" control law?



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## Nonlinear Control (cont'd)

A common approach is to use Lyapunov stability

- If the nonlinearities can be bounded,

$$|f(X)| < a$$

- Stability can be assured if the input is larger than that bound

$$(CX)^T (CAX + f(X) + CBU) < U$$

"Optimal" control is harder since our tools depend upon linear systems

- Linearize about multiple points
  - Find the optimal control for each point
  - Interpolate between optimal controllers
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# Robust Control

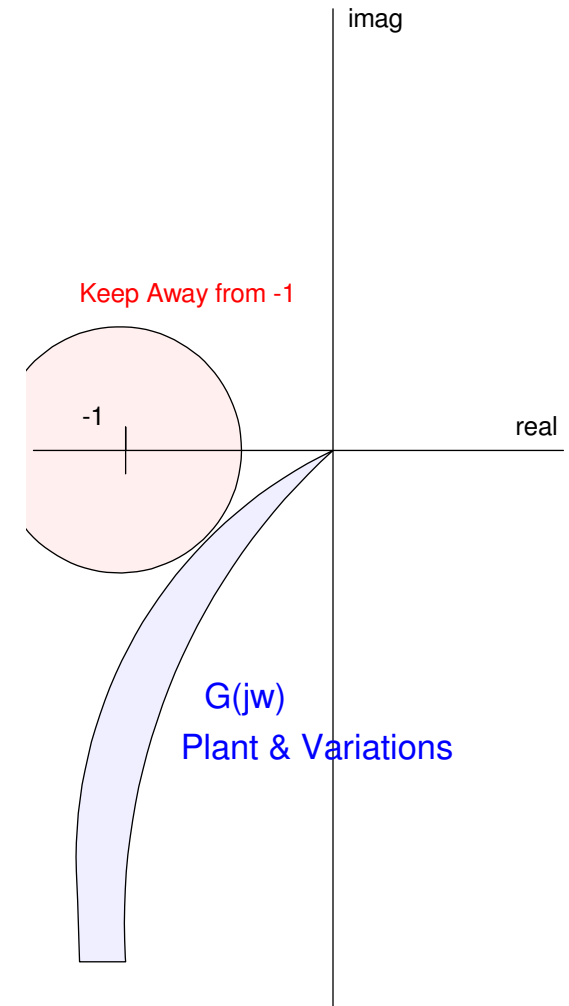
Problem: The linear model for a system actually varies

- -40C behaves different from +40C
- 0m vs. 5000m elevation has different air density, temperature humidity
- Full car vs. empty car
- New car vs. 100,000 miles

Can you design a controller that works well for a variety of systems?

Solution tends to use frequency-domain techniques

- Instability when the open-loop gain is 1.000 at 180 degrees
- Keep away from -1
- Use a generalized distance for MIMO systems



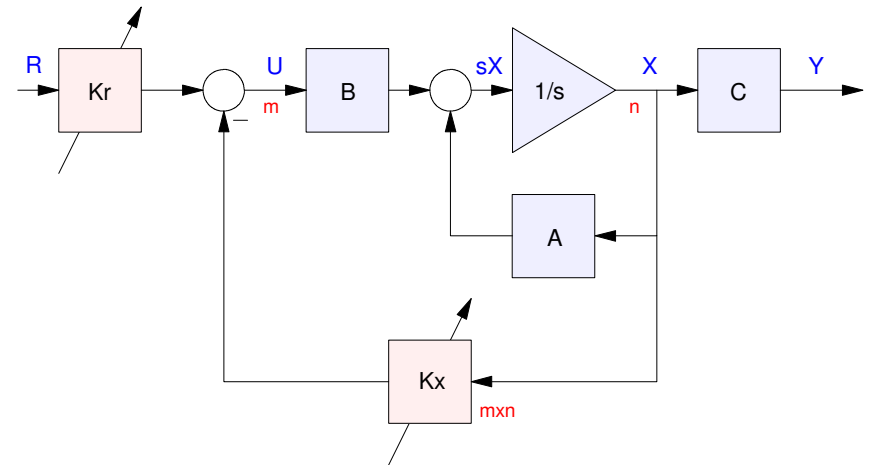
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# Adaptive Control

- Sometimes, the dynamics are unknown
- Sometimes, the dynamics are slowly time-varying

How do you adjust the control law so that the closed-loop response remains unchanged?

- In spite of not knowing the system's exact dynamics
- In spite of changes in the system's dynamics



How do you guarantee stability?

- Result is a nonlinear system
  - Limited tools for proving stability of nonlinear systems
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# Self-Tuning Regulators

- a.k.a. minimum variance control

The system's dynamics can be estimated using recursive least squares

$$y(k) = a_1y(k-1) + a_2y(k-2) + \dots + b_0u(k) + b_1u(k-1) + b_2u(k-2) + \dots$$

$u(k)$  can then be computed to drive  $y(k)$  to some desired value

$$u(k) = \frac{1}{b_0}(y(k) - a_1y(k-1) - a_2y(k-2) - \dots - b_1u(k-1) - b_2u(k-2) - \dots)$$

Applications:

- Oil Tankers: Minimize the variance in the ship's heading
- Shipping: Dynamics vary with ocean depth

Issues:

- Using recursive least squares with a forgetting factor (heavier weight on newer data)
  - Bursting: The gain goes to infinity if the input isn't sufficiently rich
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