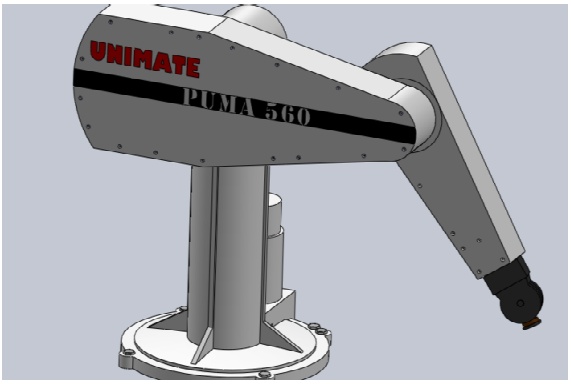
# Robot Programming

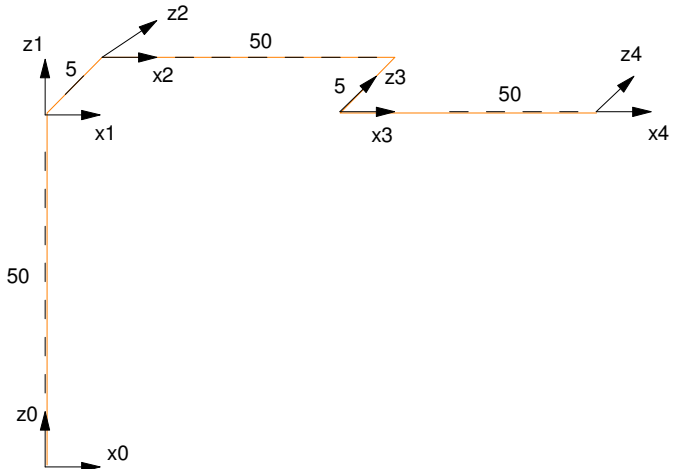
So far, we have

- Forward kinematics which allows you to determine the tip position given a set of joint angles.
- Inverse kinematics which allow you to determine the joint angles which put you at a given point.

This alone is a little inconvenient if you want to write a program which makes the robot follow a desired path. To simplify programming a robot, it would help if you wrote a set of routines for robot motion. When you combine these, you can trace out different shapes.

Start with a Puma robot:




Some useful routines would be:

```
[Q] = MoveTo(P0, P1, T)
```

Go from point P0 to point P1 in T seconds in a straight line. Use cosine interpolation so that the initial and final velocities are zero. The function returns the final position of the robot (Pf) and the joint angles every 0.01 second.

```
[Q] = Circle(P0, Pc, T)
```

Draw a circle, starting at point P0, with a center of the circle at Pc, in T seconds.

```
[Pf, Q] = ArcXY(P0, Pc, angle, T)
```

Draw an arc in the XY plane, starting at P0, with the center of the circle at Pc, at angle radians, in T seconds.

The following describes these subroutines.

## MoveTo:

Use interpolation to find points from P0 to P1 every 0.01 second

$$P_{tip} = (1-a)P_0 + (a)P_1$$

where

$$0 < a < 1$$

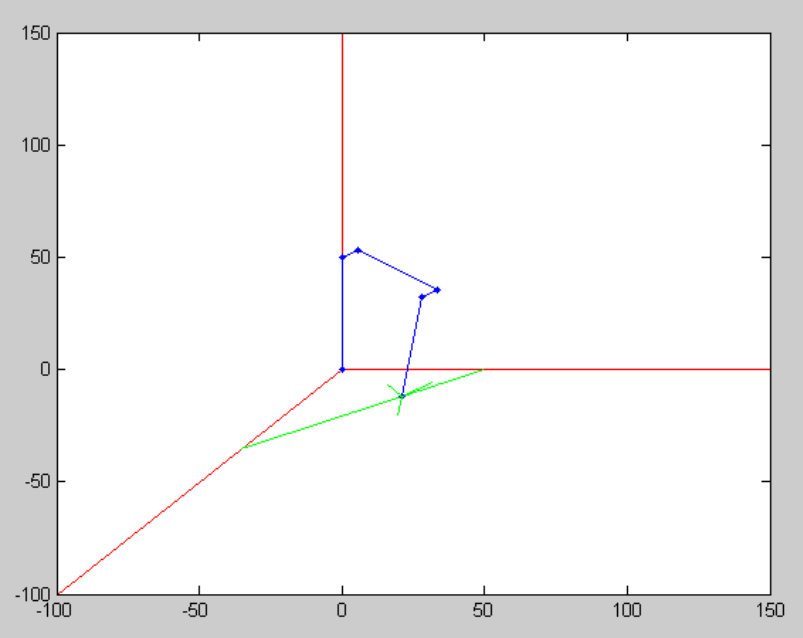
Code:

```
function [Q] = MoveTo(P0, P1, T)
% move from point P0 to P1 in T seconds

t = [0:0.01:T];
a = (1 - cos(pi*t/T))/2;

TIP = P0 * (1-a)  + P1 * a;

Q = [];

for i=1:length(TIP)
   Qi = InversePuma(TIP(:,i));
   Q = [Q, Qi];
   Puma(Qi, TIP);
   end

end
```

Example: Draw a line from [50,0,0]' to [0,50,0]' in 3 seconds.

```
P1 = [50,0,0]';
P2 = [0,50,0]';

Q = MoveTo(P1, P2, 3);
```

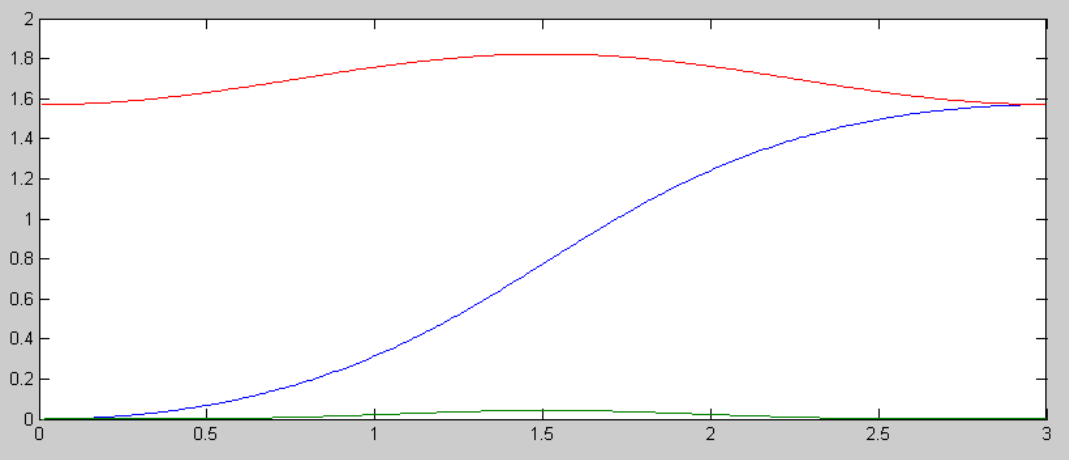
This results in the following plot:




MoveTo Command: Drawing a line from [50,0,0]' to [0,50,0]' in 3 seconds

To plot the joint angles during this motion, use

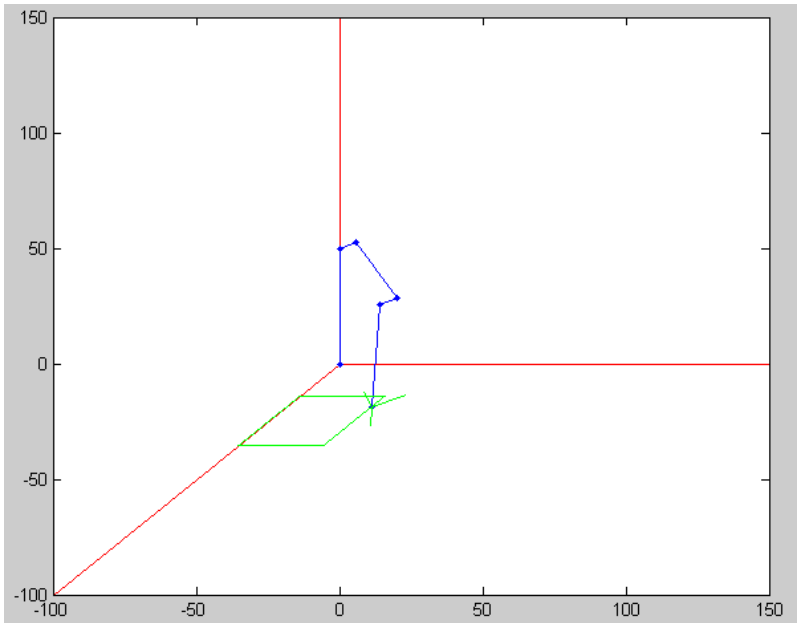
```
t = [1:length(Q)]' * 0.01;
clf
plot(t,Qi)
```

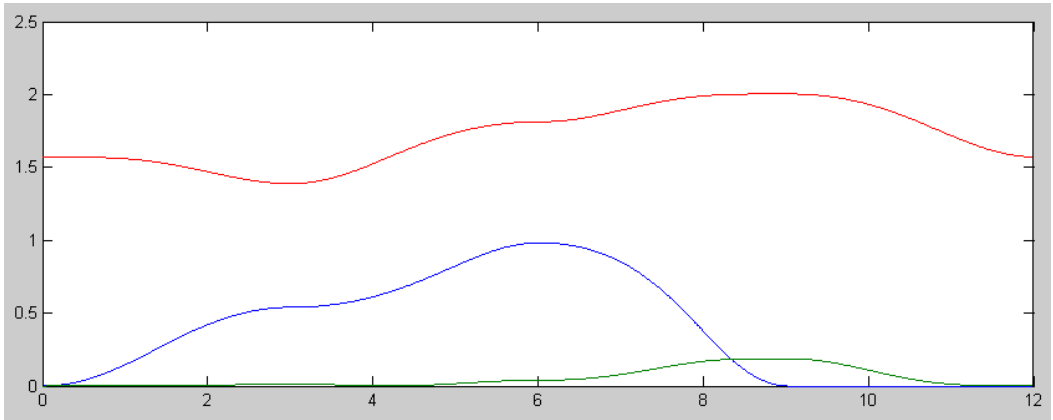



Joint Angles during the MoveTo command. Note that the initial and final velocities are zero

Example 2: Draw a box passing through the points

```
P1 = [50,0,0]';
P2 = [50,30,0]';
P3 = [20,30,0]';
P4 = [20,0,0]';

Q1 = MoveTo(P1, P2, 3);
Q2 = MoveTo(P2, P3, 3);
Q3 = MoveTo(P3, P4, 3);
Q4 = MoveTo(P4, P1, 3);

Q = [Q1, Q2, Q3, Q4];

t = [1:length(Q)]' * 0.01;
clf
plot(t,Q)
```




Joint Angles for drawing a square with MoveTo command

## CircleXY()

The equation for a circle is

$$x = x_0 + r \cdot \cos\theta$$

$$y = y_0 + r \cdot \sin\theta$$

where

- $(x_0, y_0)$ is the center of the circle
- r is the radius, and
- $0 < \theta < 2\pi$

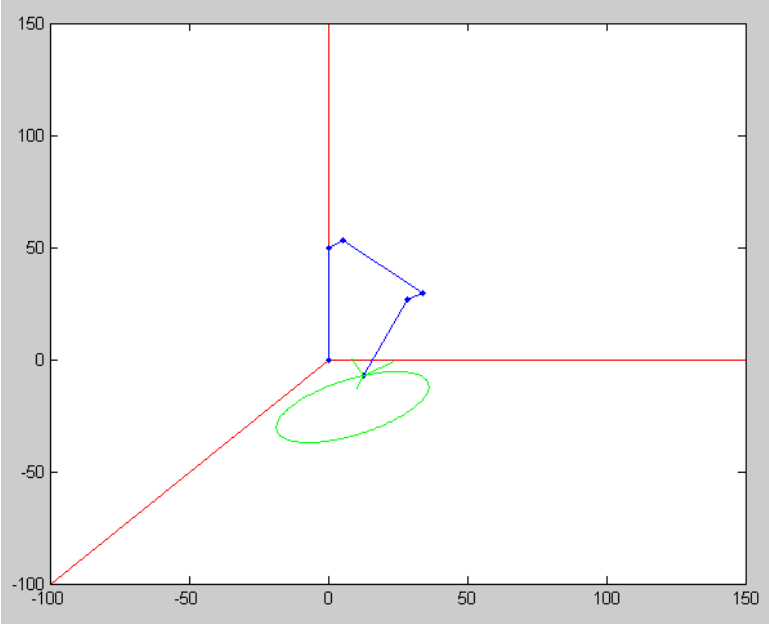
To start at a specific point on the circle, add a phase shift to $\theta$.

Code:

```
function [P0, TIP] = CircleXY(P0, P1, T)

% Draw a cirle in the XY plane with center at P1
% starting at point P0
% in T seconds
```

Example: Draw a circle, starting at point [10,20,30]', centered at [30,30,30]', in 3 seconds

```
>> [Pf, Q] = CircleXY([10,20,0]',  [30,30,0]',  3);
```




Result from drawing a circle, starting at point [10,20,30]', centered at [30,30,30]', in 3 seconds

## ArcXY(P1, P2, P3, T)

Draw an arc in the XY plane, starting from point P1, passing through point P2, ending at point P3, in T seconds.

The mathematics for this is as follows. Any point on a circle is defined as

$$(x-x_0)^2+(y-y_0)^2=r^2$$

Plugging in points for P1, P2, P3 results in three equations for three unknowns (x0, y0, r).

$$(x_1-x_0)^2+(y_1-y_0)^2=r^2$$
$$(x_2-x_0)^2+(y_2-y_0)^2=r^2$$
$$(x_3-x_0)^2+(y_3-y_0)^2=r^2$$

Set equation 1 equal to the second and third results in two equations for two unknowns (x0, y0)

$$(x_1-x_0)^2+(y_1-y_0)^2=r^2=(x_2-x_0)^2+(y_2-y_0)^2$$
$$(x_1-x_0)^2+(y_1-y_0)^2=r^2=(x_3-x_0)^2+(y_3-y_0)^2$$

Multiply these out

$$\left(x_1^2-2x_1x_0+x_0^2\right)+\left(y_1^2-2y_1y_0+y_0^2\right)=\left(x_2^2-2x_2x_0+x_0^2\right)+\left(y_2^2-2y_2y_0+y_0^2\right)$$
$$\left(x_1^2-2x_1x_0+x_0^2\right)+\left(y_1^2-2y_1y_0+y_0^2\right)=\left(x_3^2-2x_3x_0+x_0^2\right)+\left(y_3^2-2y_3y_0+y_0^2\right)$$

Simplify

$$\left(x_1^2-2x_1x_0\right)+\left(y_1^2-2y_1y_0\right)=\left(x_2^2-2x_2x_0\right)+\left(y_2^2-2y_2y_0\right)$$
$$\left(x_1^2-2x_1x_0\right)+\left(y_1^2-2y_1y_0\right)=\left(x_3^2-2x_3x_0\right)+\left(y_3^2-2y_3y_0\right)$$

Group terms

$$\begin{bmatrix}(2x_2-2x_1) & (2y_2-2y_1)\\ (2x_3-2x_1) & (2y_3-2y_1)\end{bmatrix}\begin{bmatrix}x_0\\ y_0\end{bmatrix}=\begin{bmatrix}\left(x_2^2+y_2^2\right)-\left(x_1^2+y_1^2\right)\\ \left(x_3^2+y_3^2\right)-\left(x_1^2+y_1^2\right)\end{bmatrix}$$

Solve for the center of the circle (x0, y0). The radius is from any of these equations:

$$r^2=(x_1-x_0)^2+(y_1-y_0)^2$$

The initial and final angle are from

$$\theta_1=\arctan\left(\frac{y_1-y_0}{x_1-x_0}\right)$$

$$\theta_3=\arctan\left(\frac{y_3-y_0}{x_3-x_0}\right)$$

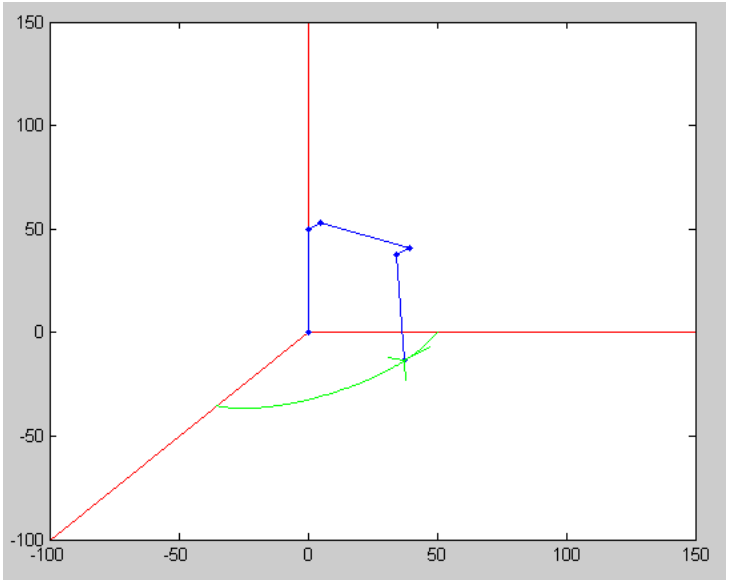
The arc goes from these two angles, with $2\pi$ added or subtracted to keep from going the long way around the circle if the angles go past zero (i.e. an arc from 350 degrees to 10 degrees should really go from -10 to +10 degrees.).

Code:

```
function [Q] = ArcXY(P1, P2, P3, T)
% Draw an arc in the XY plane from P1 to P3 passing through P2
% in T seconds
% rev 5/26/15
```

Sample Code: Draw a

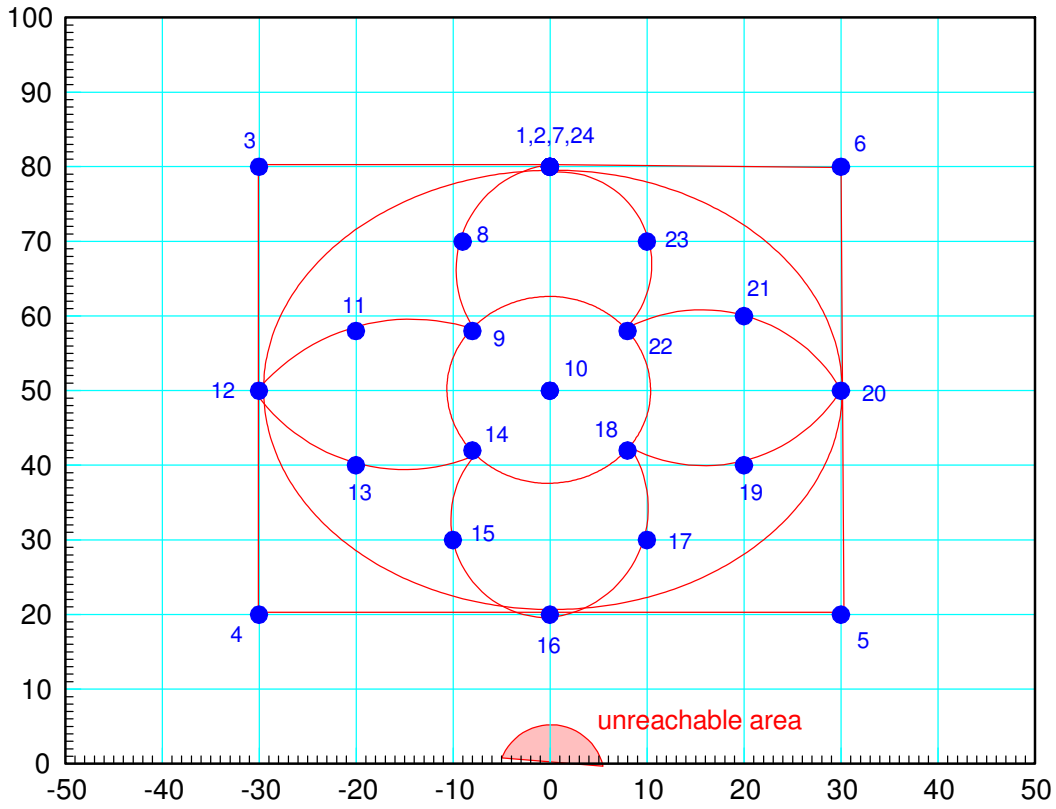
```
[Pf, TIP] = ArcXY([50,0,0]',  [40,40,0]',  [0,50,0]', 3);
```



## Draw a Dogwood Symbol:

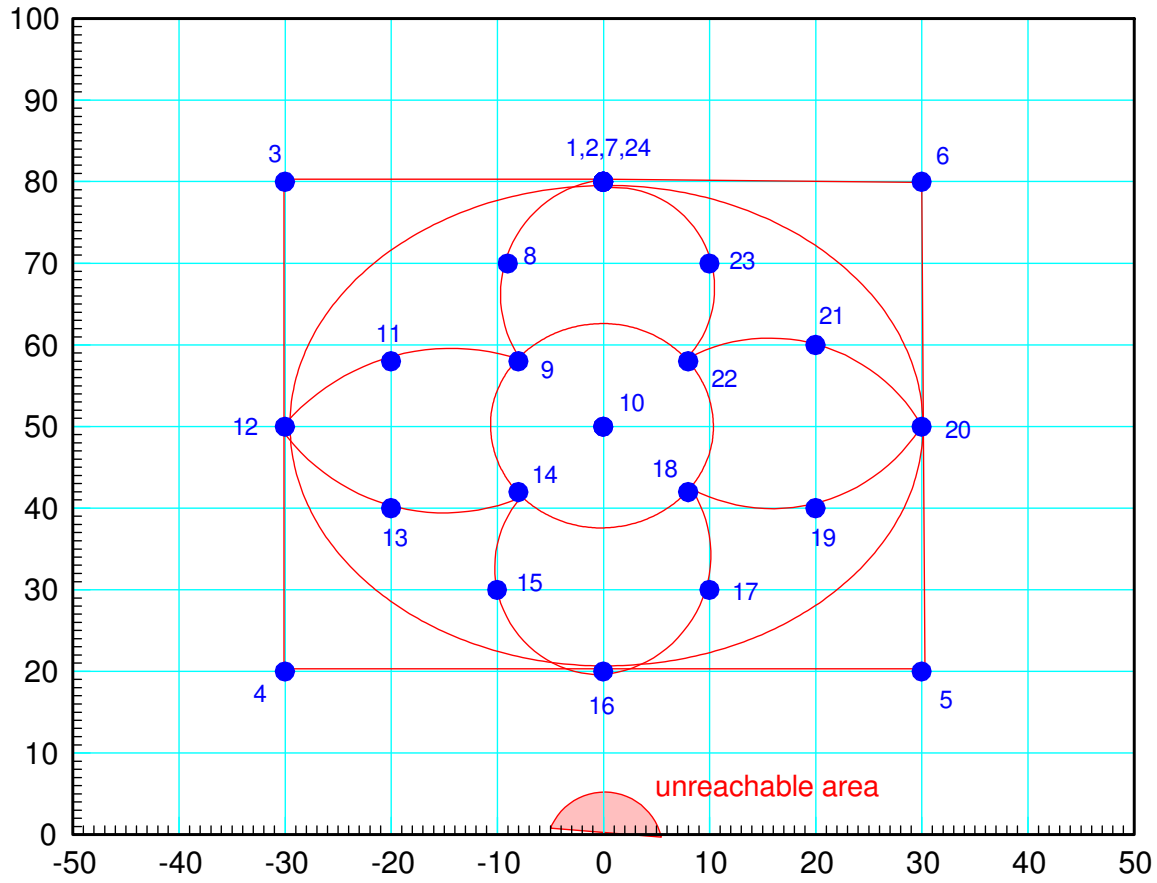
First, define the points for your shape

```
P1 = [0; 80; 20];
P2 = [0; 80; 0];
P3 = [-30; 80; 0];
P4 = [-30; 20; 0];
P5 = [30; 20; 0];
P6 = [30; 80; 0];
P7 = [0; 80; 0];
P8 = [-9; 70; 0];
P9 = [-8; 58; 0];
P10 = [0; 50; 0];
P11 = [-20; 58; 0];
P12 = [-30; 50; 0];
P13 = [-20; 40; 0];
P14 = [ -8; 42; 0];
P15 = [-10; 30; 0];
P16 = [  0; 20; 0];
P17 = [ 10; 30; 0];
P18 = [  8; 42; 0];
P19 = [ 20; 40; 0];
P20 = [ 30; 50; 0];
P21 = [ 20; 60; 0];
P22 = [  8; 58; 0];
P23 = [ 10; 70; 0];
P24 = [  0; 80; 0];
P25 = [0; 80; 20];
```



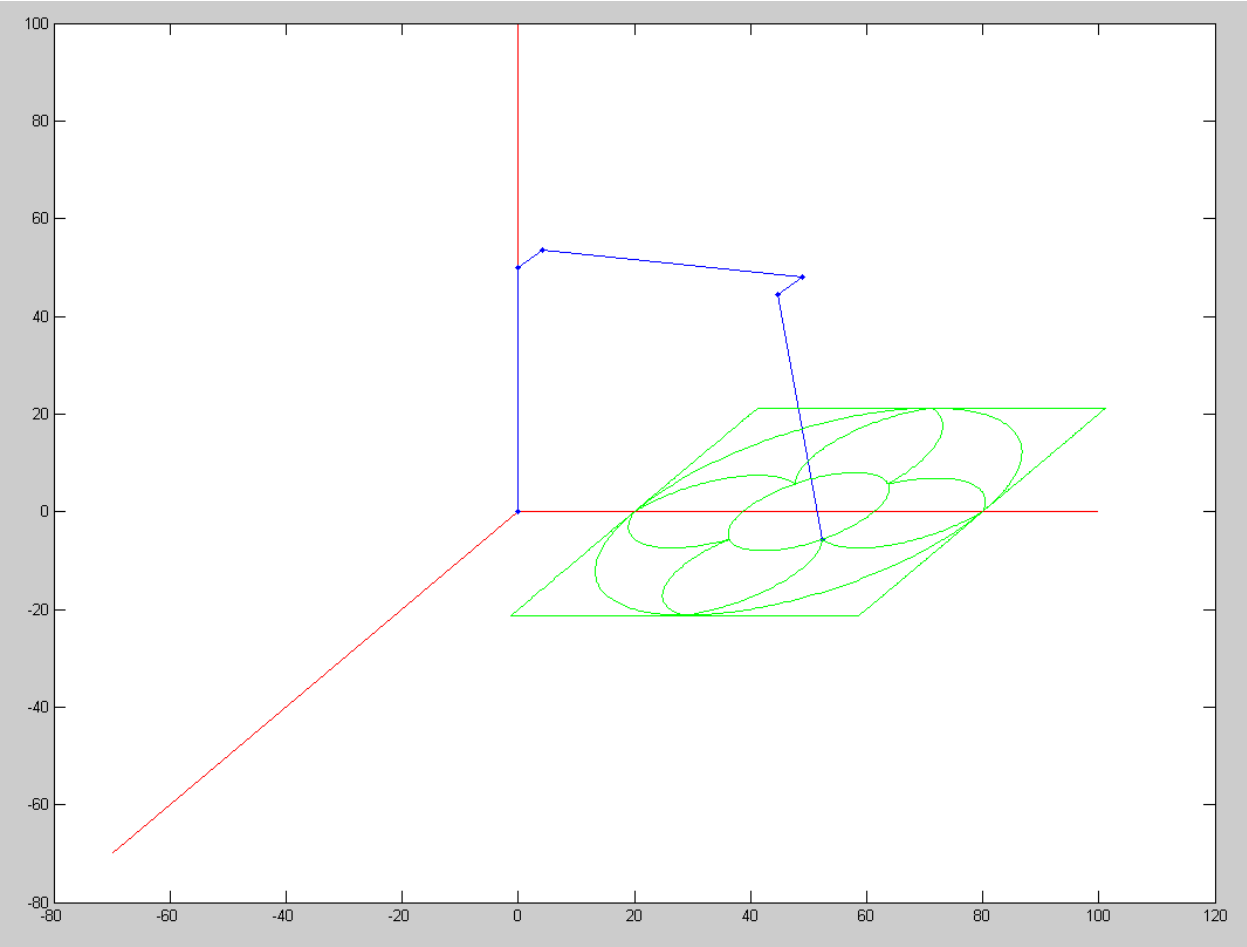
Next, write a bunch of move / circle / arc commands

```
% pen down
Q1 = MoveTo(P1, P2, 1);
% draw a box
Q2 = MoveTo(P2, P3, 2);
Q3 = MoveTo(P3, P4, 2);
Q4 = MoveTo(P4, P5, 2);
Q5 = MoveTo(P5, P6, 2);
Q6 = MoveTo(P6, P7, 2);
% draw a circle
Q7 = CircleXY(P7, P10, 4);
% draw arcs
Q8 = ArcXY(P7, P8, P9, 1);
% draw a circle
Q9 = CircleXY(P9, P10, 2);
% draw arcs
Q10 = ArcXY(P9, P11, P12, 1);
Q11 = ArcXY(P12, P13, P14, 1);
Q12 = ArcXY(P14, P15, P16, 1);
Q13 = ArcXY(P16, P17, P18, 1);
Q14 = ArcXY(P18, P19, P20, 1);
Q15 = ArcXY(P20, P21, P22, 1);
Q16 = ArcXY(P22, P23, P24, 1);
% pen up
Q17 = MoveTo(P24, P1, 1);
Q = [Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17];
```

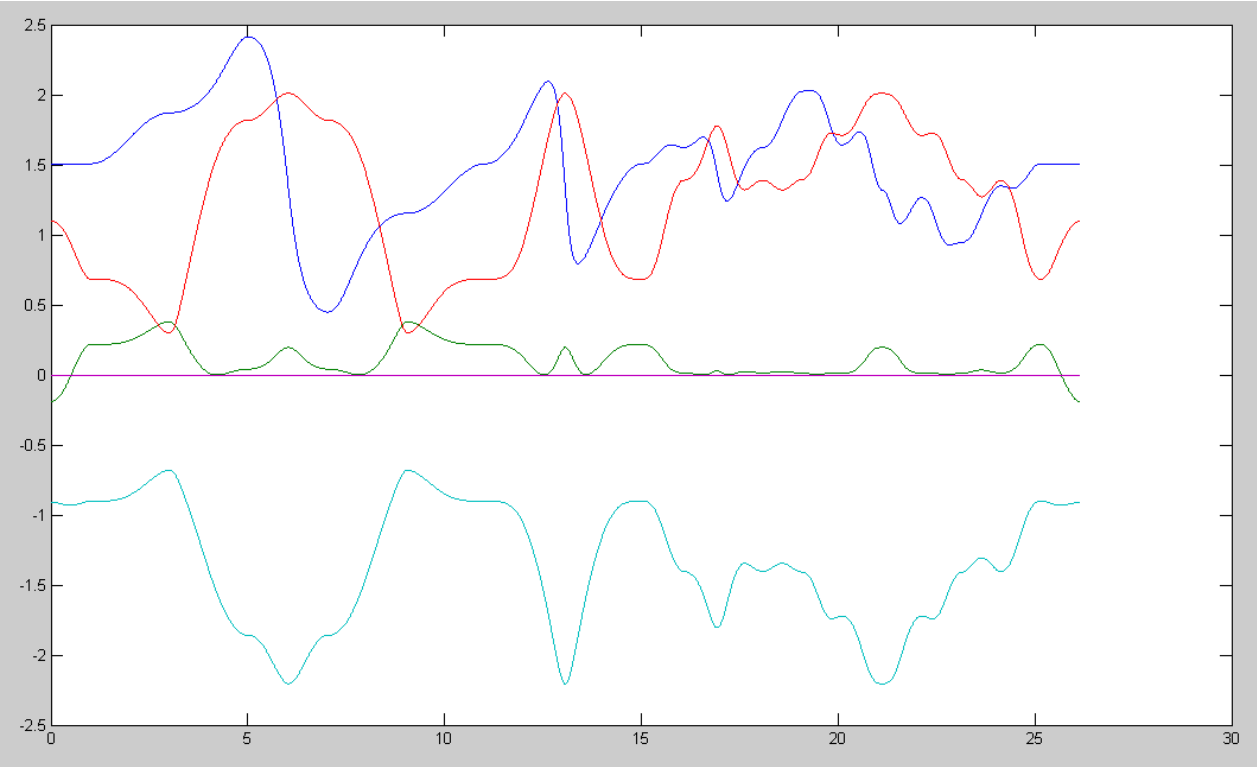


The angle matrix (Q) is the program for drawing the dogwood (the angles for the robot every 10ms)

Result




The joint angles are then:




Joint angles vs. time for drawing a dogwood flower