Forward Kinematics

Lecture #4

ECE 761: Robotics

Class taught at North Dakota State University
Department of Electrical and Computer Engineering

Please visit www.BisonAcademy.com for corresponding lecture notes, homework sets, and solutions.

Forward Kinematics

Given the joint angles, determine the tip position

To do this, assign a reference frame to each joint

- First, you rotate the reference frame about the x-axis so that the two z-axis line up (twist)
- Second, move along the x axis
- Third, rotate about the z-axis, finally
- Move along the z-axis to end up at the next reference frame.

This defines the translation matrix from joint i-1 to joint i

Translation Matrix:

$$T_{01} = R_x(\alpha_0) D_x(a_0) R_z(\theta_1) D_z(d_1)$$

or, multiplying it out:

$$T_{01} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_0 \\ \sin\theta_1 c \cos\alpha_0 & \cos\theta_1 \cos\alpha_0 & -\sin\alpha_0 & -\sin\alpha_0 \, d_1 \\ \sin\theta_1 \sin\alpha_0 & \cos\theta_1 \sin\alpha_0 & \cos\alpha_0 & \cos\alpha_0 \, d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is the subroutine Transform.m posted on Bison Academy.

Example: Determine the transformation matrix T01 when

- alpha0 = 90 degrees (twist)
- a0 = 50cm
- d1 = 40cm
- theta1 = 30 degrees (rotation)

Solution:

```
Transform(pi/2, 50, 40, pi/4)

    0.7071   -0.7071         0   50.0000
    0.0000    0.0000   -1.0000  -40.0000
    0.7071    0.7071    0.0000    0.0000
         0         0         0    1.0000
```

Repeating this procedure for joints 1 to 2, 2 to 3, etc. allows you to find the transformation matrix to go from tip coordinates to base (earth) coordinates.

Types of Joints:

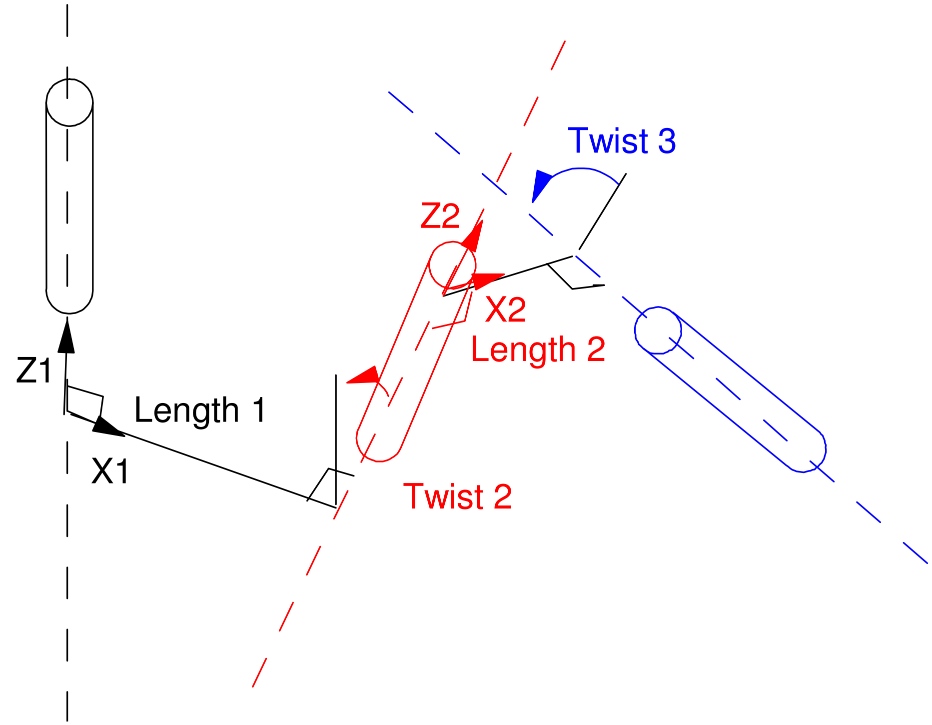
- Translational: The motor changes the length of the arm
- Rotational: The motor changes the angle of the arm
- Compound: If a joint can do several things, like a hip joint

If a joint is compound, it is treated as two separate joints with a displacement of zero.

Link Numbering:

Start at 0 (earth reference)

- Each subsequent reference frame is numbered from 1 on up.
- Link i connects axis i to i+1
- **Length i:** The length of link i is the distance from axis i to i+1. This is the shortest distance between the two lines that pass through the axis of rotation.
- **Twist i:** Relative to the line which is perpendicular to both axis (i.e. looking down this line), the twist is the angle between the two axis.



Procedure:

1. Identify the joint axis.   Draw an infinite line along each axis.

2. Identify the perpendicular lines between each axis. At the point of intersection, define a reference frame.

3. Assign the Zi axis pointing along the ith joint axis.

4. Assign the Xi axis pointing to the next axis (i.e. along the perpendicular line between axis). If the joint axis intersect, Xi is perpendicular to both Z axis.

5. Assign the Yi axis following the right-hand rule

$$X \times Y = Z$$

6. Assign axis 0 to match axis 1 when the first joint angle is zero.

7. The zero position is when a all X axis are pointing in the same direction.

Link Parameters:

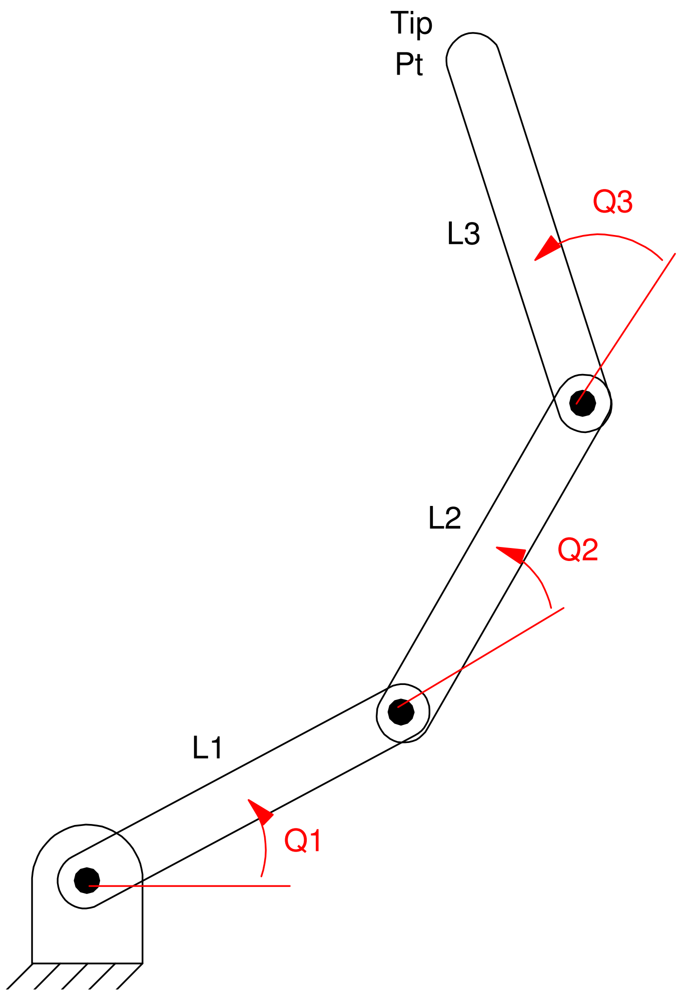
a_i The distance from Zi to Zi+1 measured along the Xi axis

α_i The angle between the Zi and Zi+1 axis (twist)

d_i The distance from Xi-1 to Xi measured along the Zi axis

θ_i The angle between Xi-1 and Xi measured about the Zi axis

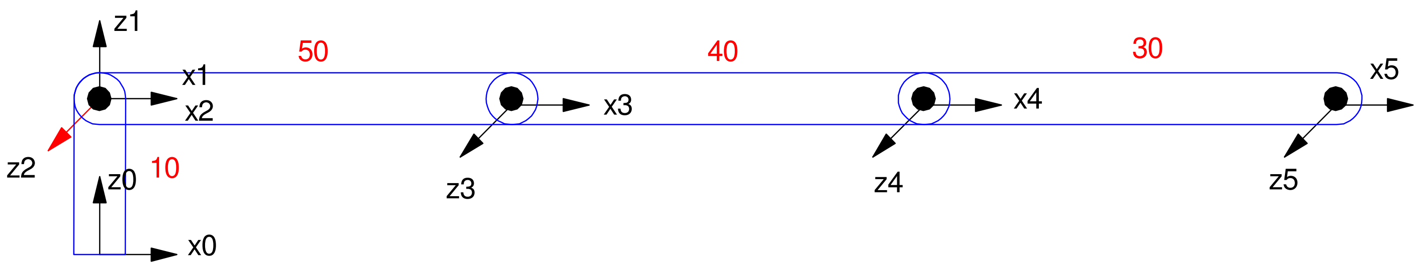
Example: Define the reference frames for the following robot (a model for a human finger):



Example 1: Modeling a human finger

Solution in zero-position: Note that if assigned correctly

- You can only move in the x and z direction. If you try to move in the y-diretion, you made a mistake in assigning the reference frames.
- In zero position, all of the x-axis point in the same direction



Link i	α_{i-1} The angle between the Zi-1 and Zi axis (twist)	a_{i-1} The distance from Zi-1 to Zi measured along the Xi-1 axis	d_i The distance from Xi-1 to Xi measured along the Zi axis	θ_i The angle between Xi-1 and Xi measured about the Zi axis
1	0	0	10	0
2	+90 degrees	0	0	θ_1
3	0	50	0	θ_2
4	0	40	0	θ_3
5	0	30	0	0

Once you define the robot and the joint angles, you can

- draw the robot and
- determine the tip position
- Program RRR.m

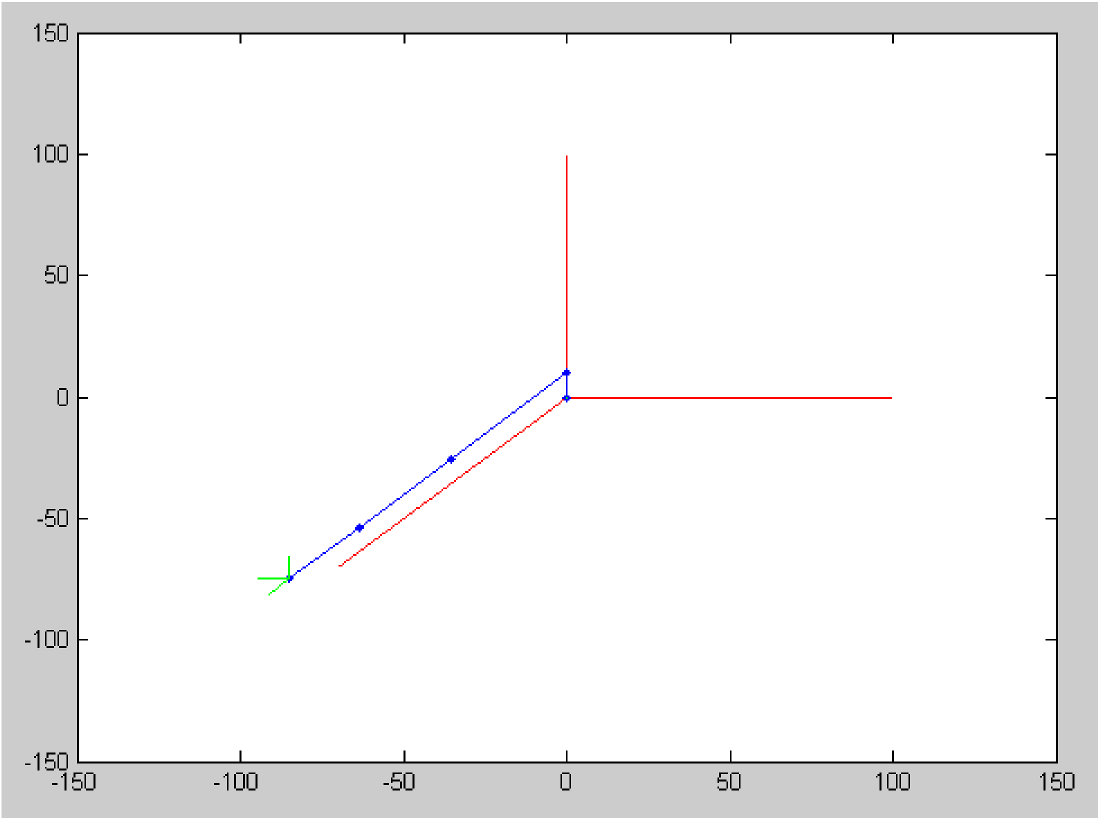
```
function [Tip] = RRR(W, TIP)

alpha = [0, pi/2,  0,   0,   0];
a     = [0,    0,  50, 40,  30];
d     = [10,   0,   0,  0,   0];
Q     = [0, W(1), W(2), W(3), 0];

T01a = Transform(alpha(1), a(1), 0,    Q(1));
T01  = Transform(alpha(1), a(1), d(1), Q(1));
T12a = Transform(alpha(2), a(2), 0,    Q(2));
T12  = Transform(alpha(2), a(2), d(2), Q(2));

etc.
```

The way this program works is, once you define the refrence frames (alpha, a, d, Q), it

- Defines the transformation matrix to reference frame #1,
- Then #2, etc.
- It then draws a line from each reference frame to the next, and
- It returns the tip position.

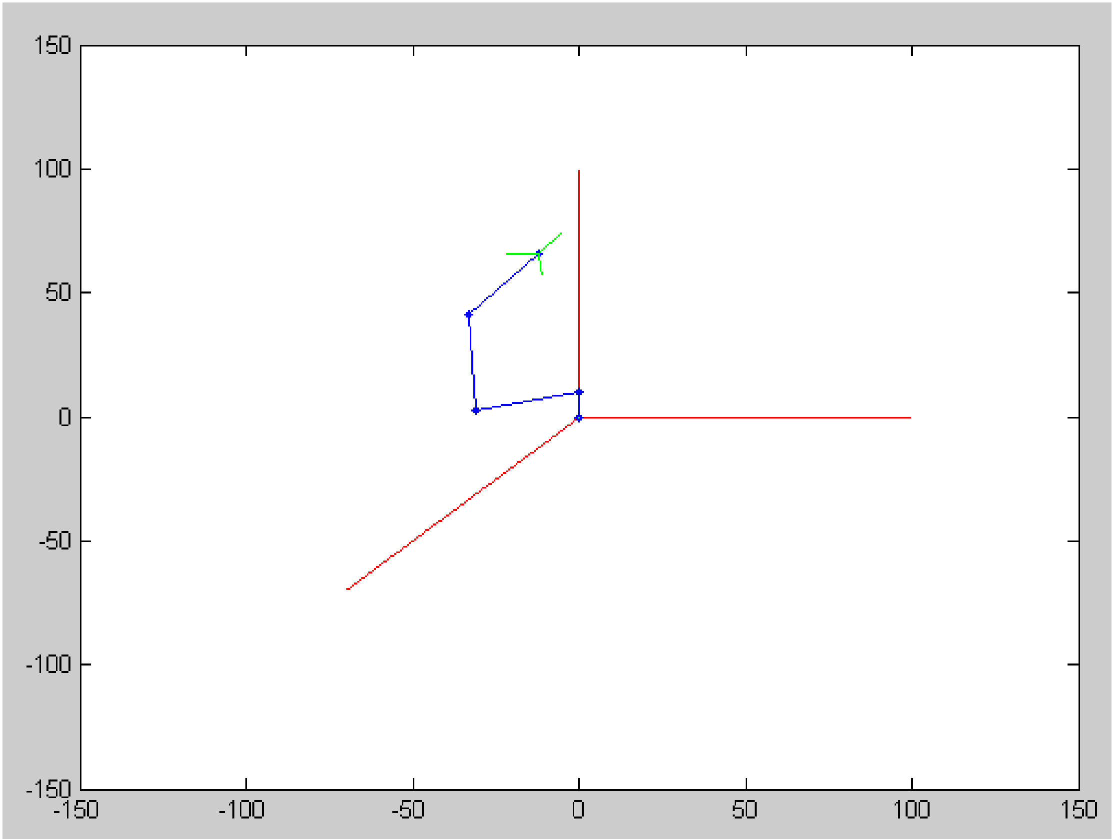
For example, the tip position in zero position is:

```
TIP = zeros(3,1);  % we will use this later
RRR([0,0,0], TIP)
x   120
y     0
z    10
      1
```



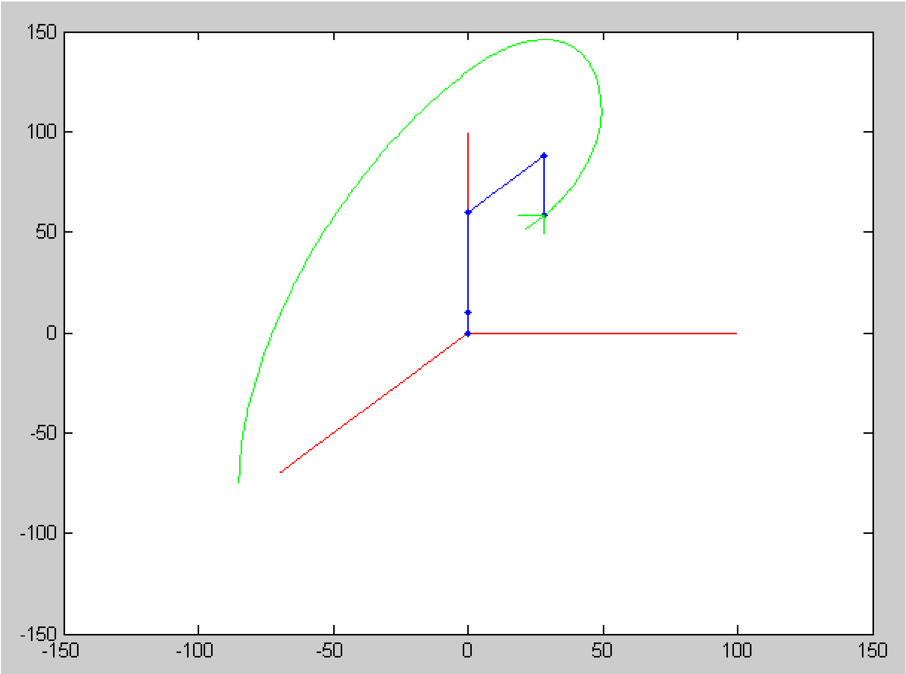
Draw the robot and calculate the tip position for angles of (0.5, 1, 1.5 radians)

```
RRR([0.5,1,1.5],TIP)

x   17.0088
y    0.0000
z   78.1047
     1.0000
```



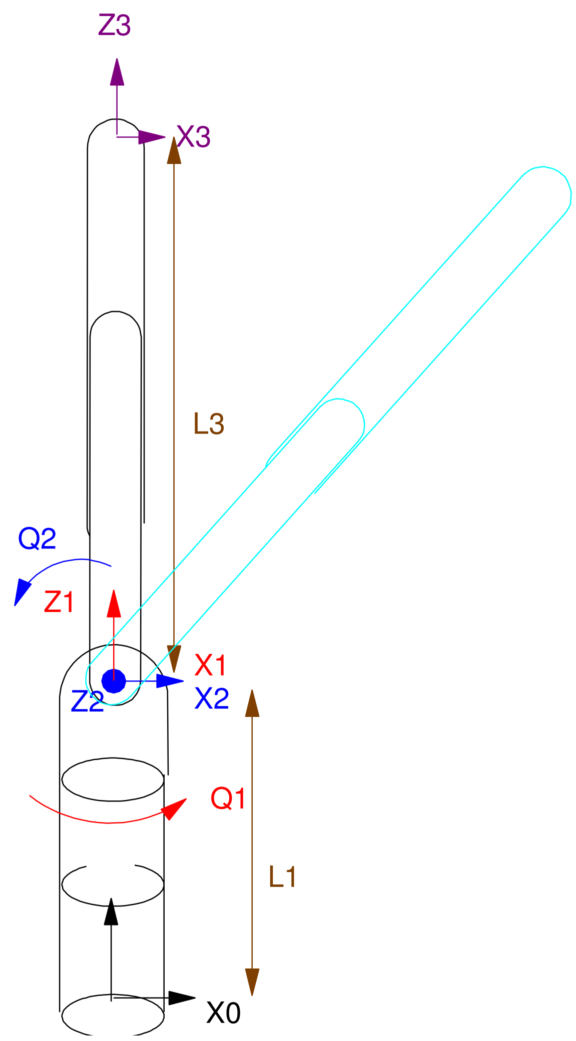
You can also show the path of the tip during a move:

- Start from zero position
- Rotate Q1 to 90 degrees, then
- Rotate Q2 to 90 degrees, then
- Rotate Q3 to 90 degrees

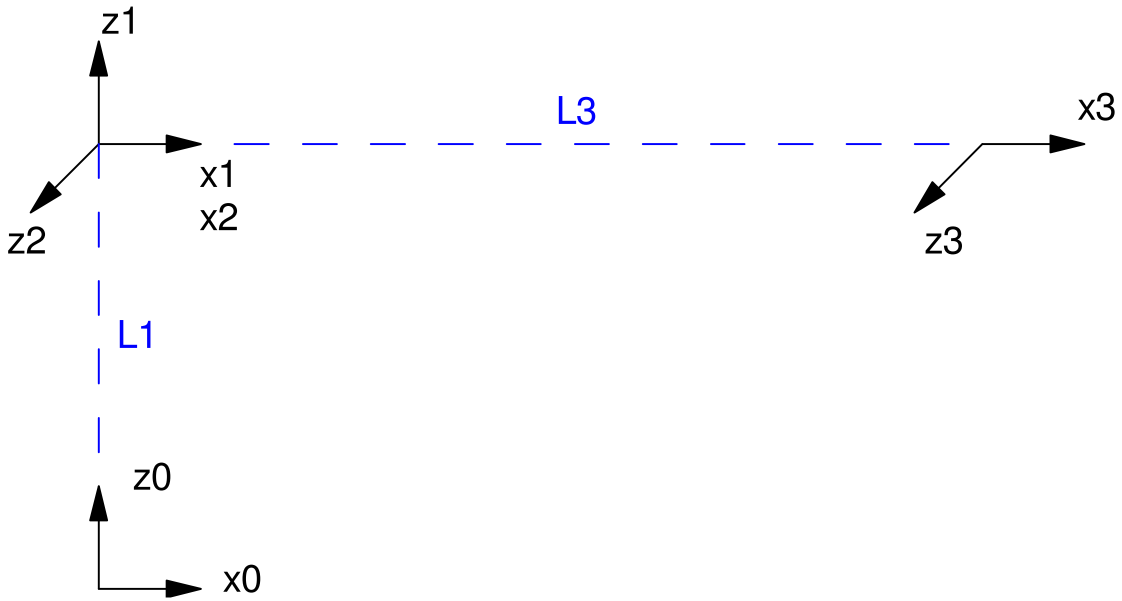



```
TIP = RRR([0,0,0], zeros(4,1));
for i=1:90
    T = RRR([i*pi/180, 0, 0], TIP);
    TIP = [TIP,T];
end

for i=1:90
    T = RRR([pi/2, i*pi/180, 0], TIP);
    TIP = [TIP,T];
end

for i=1:90
    T = RRR([pi/2, pi/2, i*pi/180], TIP);
    TIP = [TIP,T];
end
```

Example 2: RRP Robot



First, redraw the robot in zero position, assigning reference frames



Reference frame definitions at zero position

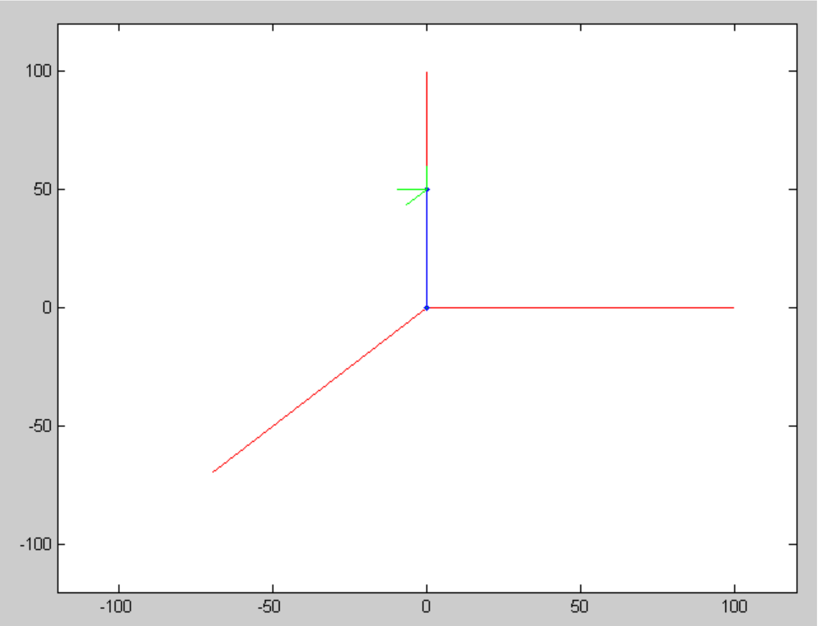
Link i	alpha $_{i-1}$ The angle between the Zi-1 and Zi axis (twist)	a_{i-1} The distance from Zi-1 to Zi measured along the Xi-1 axis	d_i The distance from Xi-1 to Xi measured along the Zi axis	Qi The angle between Xi-1 and Xi measured about the Zi axis
1	0	0	L1	Q1
2	+90 degrees	0	0	Q2
3	0	L3	0	0

To simulate this robot, change the first four lines of code in RRR (creating RRP)

```
function [Tip] = RRP(W, TIP)

alpha = [0, pi/2,    0,  0,  0];
a     = [0,    0, W(3),  0,  0];
d     = [50,   0,    0,  0,  0];
Q     = [W(1), W(2), 0,  0,  0];

etc.
```

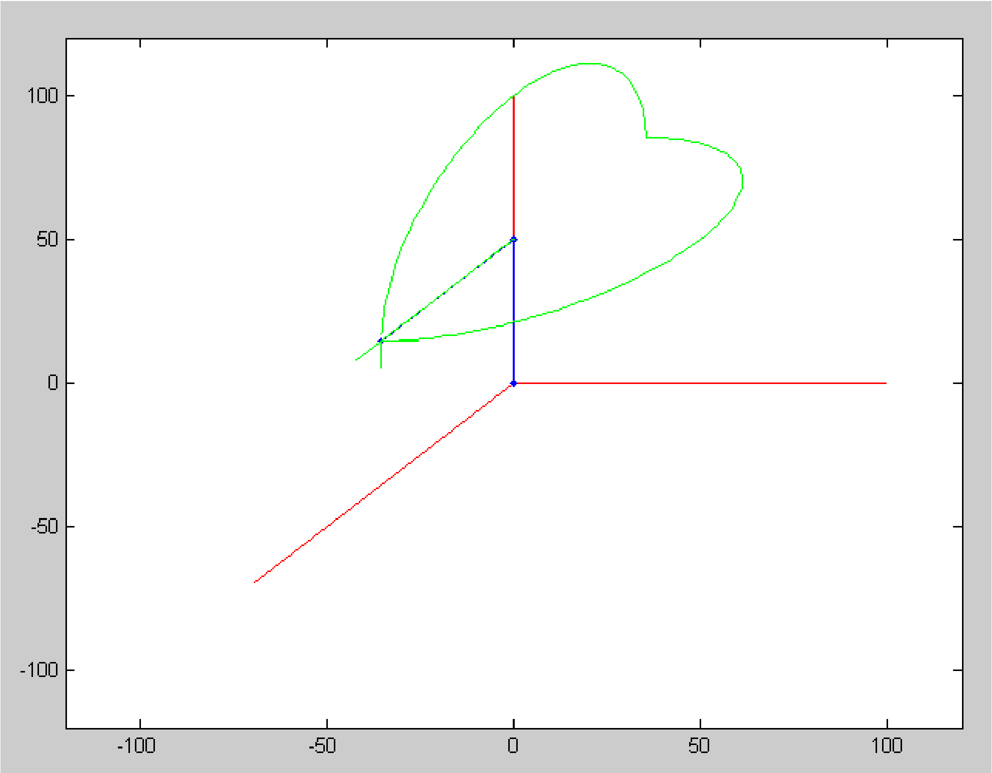
Determine the tip position at zero position:

```
>> TIP = RRP([0,0,0],zeros(4,1))

x      0
y      0
z     50
       1


TIP = RRP([pi/4, pi/3, 50],zeros(4,1))

x   17.6777
y   17.6777
z   93.3013
     1.0000
```



Trace out the tip position as

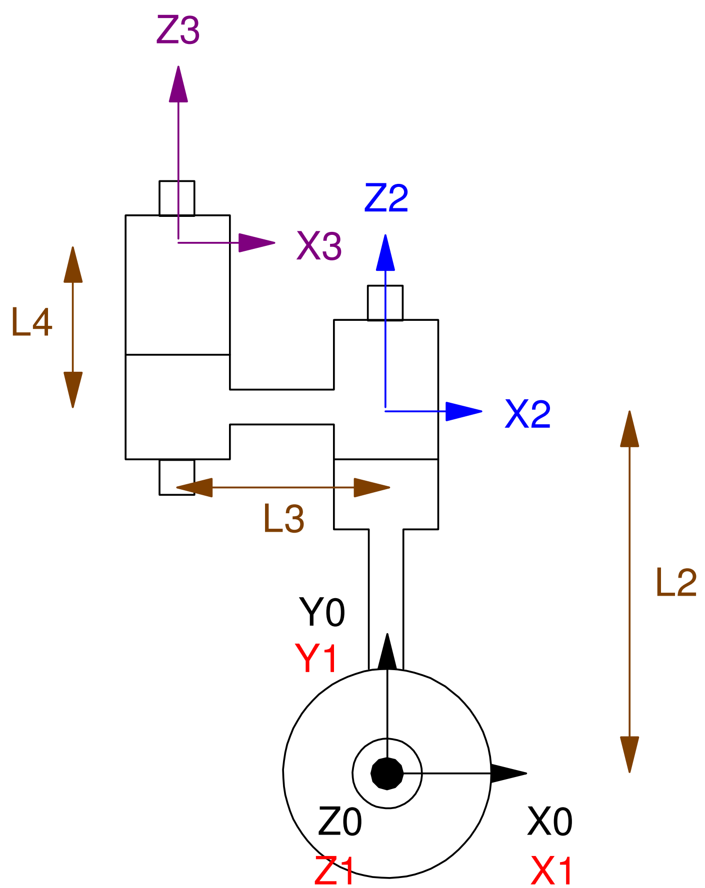
- L3 goes from 0 to 50cm, then
- Q1 goes from 0 to 180 degrees, then
- Q2 goes from 0 to 180 degrees



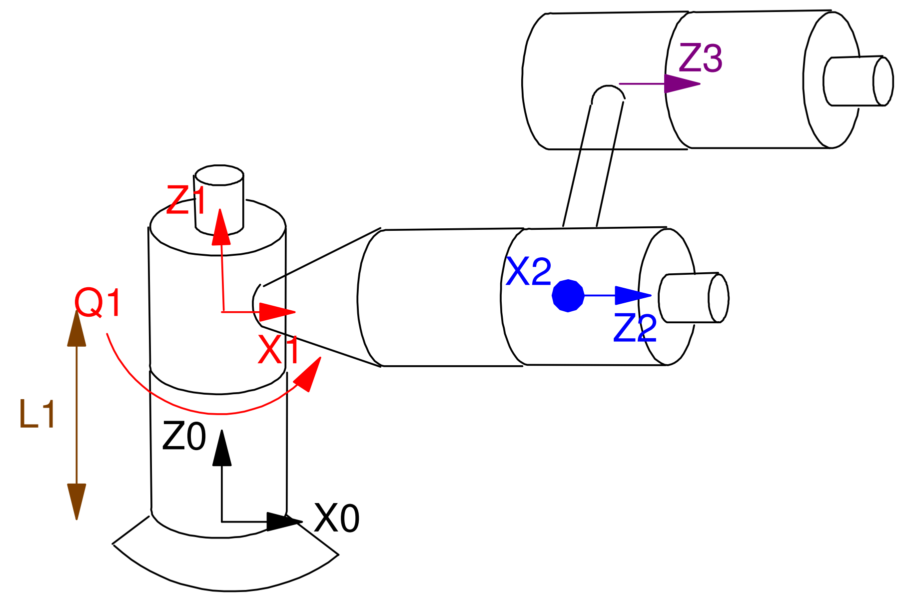
Code

```
TIP = RRP([0,0,0], zeros(4,1));
for i=1:50
    T = RRP([0, 0, i], TIP);
    TIP = [TIP,T];
end
for i=1:90
    T = RRP([i*pi/90, 0, 50], TIP);
    TIP = [TIP,T];
end
for i=1:90
    T = RRP([pi, i*pi/90, 50], TIP);
    TIP = [TIP,T];
end
```

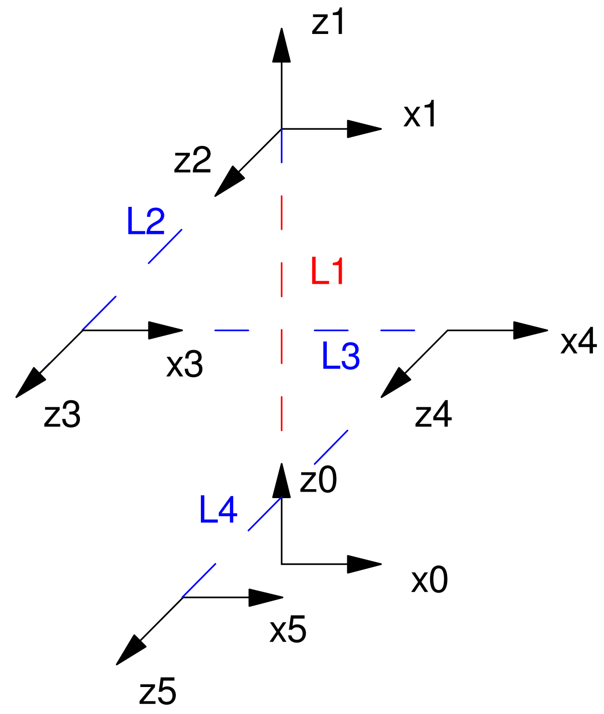
Example 3: Another RRR Robot (RRR3)



Top View - Zero Position

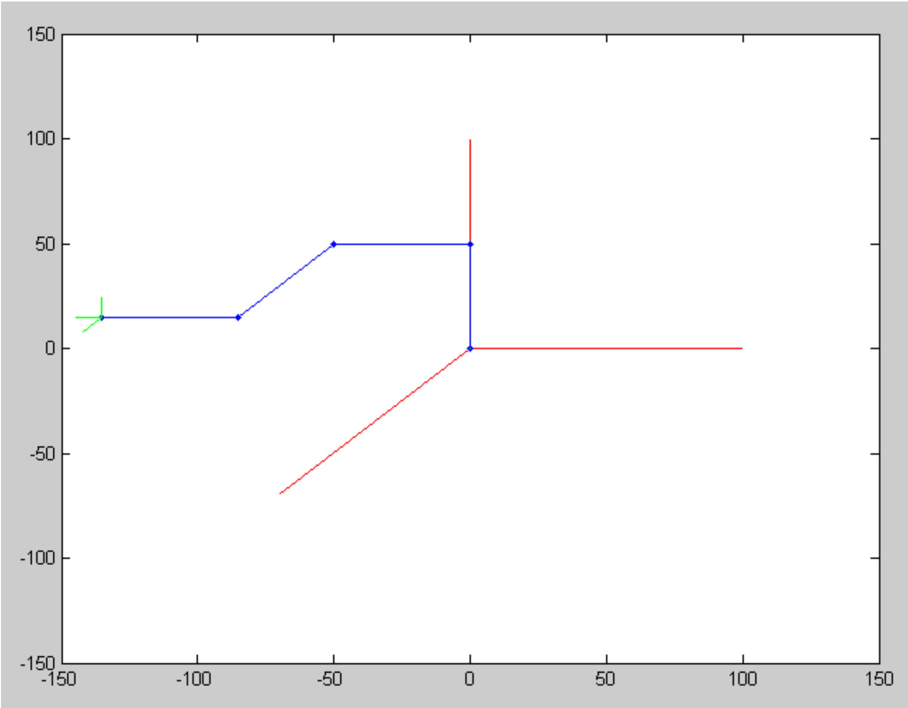


Link i	alpha $_{i-1}$ The angle between the Zi-1 and Zi axis (twist)	a_{i-1} The distance from Zi-1 to Zi measured along the Xi-1 axis	d_i The distance from Xi-1 to Xi measured along the Zi axis	Qi The angle between Xi-1 and Xi measured about the Zi axis
1	0	0	L1	Q1
2	pi/2	0	0	0
3	0	0	L2	Q2
4	0	L3	0	0
5	0	0	L4	Q3



In zero position, the robot is at

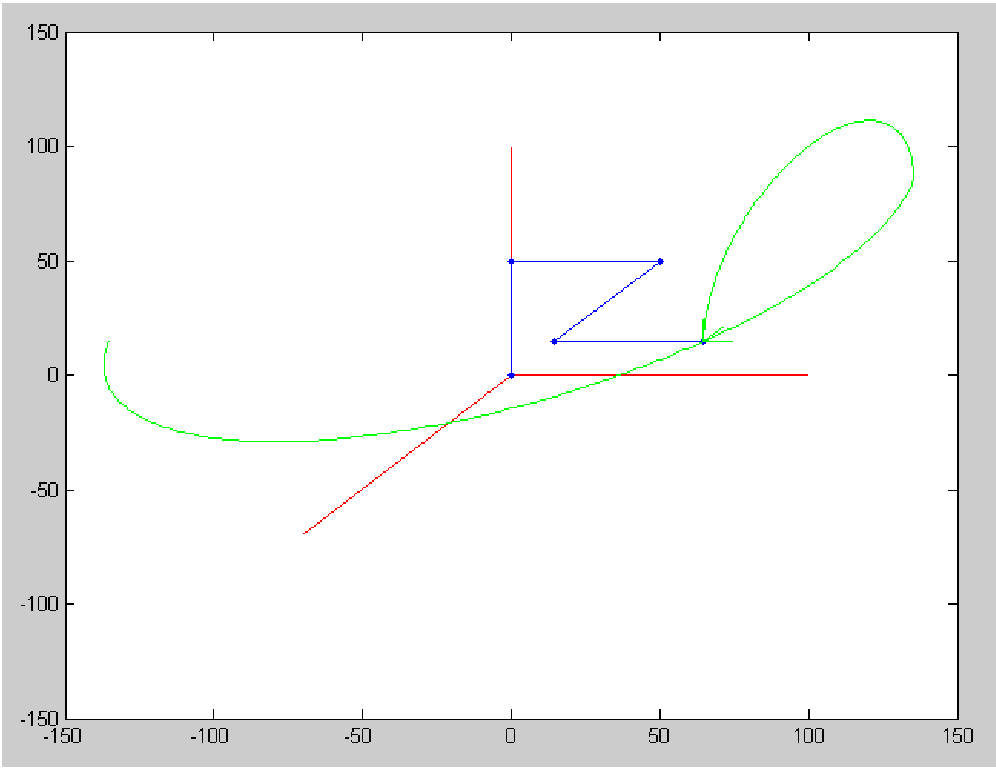
```
RRR3([0,0,0],zeros(4,1))

    50
  -100
    50
     1
```



Zero position for the RRRv3 robot

Moving each joint:

- Q1 goes from 0 to 180 degrees, then
- Q2 goes from 0 to 180 degrees, then
- Q3 goes from 0 to 180 degrees



Code:

```
TIP = RRR3([0,0,0], zeros(4,1));
for i=1:180
    T = RRR3([i*pi/180, 0, 0], TIP);
    TIP = [TIP,T];
end
for i=1:180
    T = RRR3([pi, i*pi/180, 0], TIP);
    TIP = [TIP,T];
end
for i=1:180
    T = RRR3([pi, pi, i*pi/180], TIP);
    TIP = [TIP,T];
end
```