Inverse Kinematics for a Rhino Robot

Lecture #5

ECE 761: Robotics

Class taught at North Dakota State University
Department of Electrical and Computer Engineering

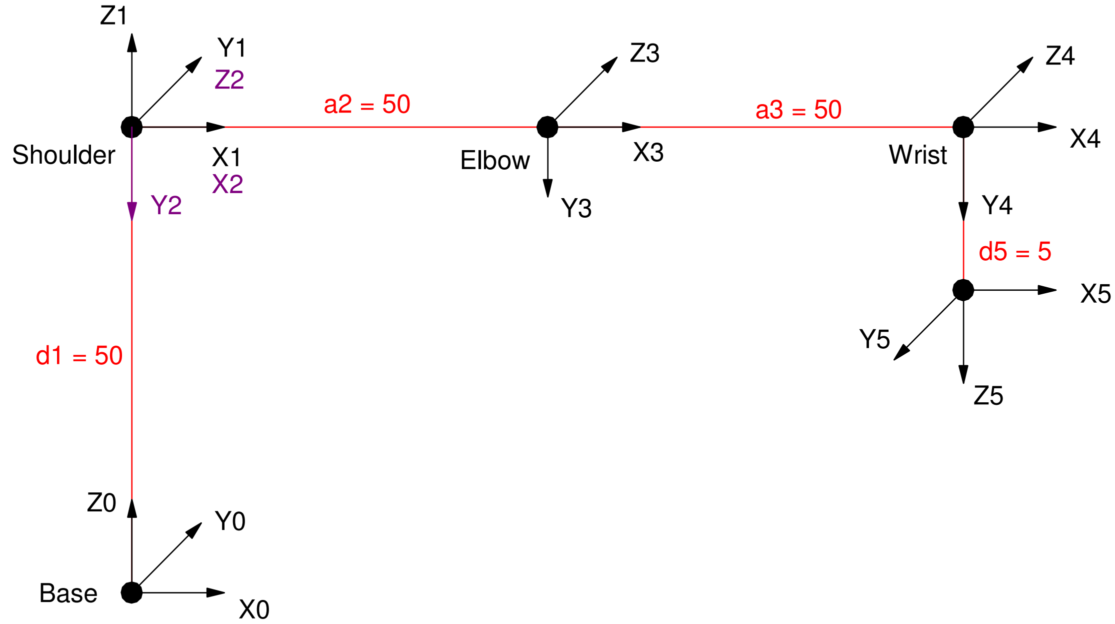
Please visit www.BisonAcademy.com for corresponding lecture notes, homework sets, and solutions.

Rhino Robot: Forward Kinamatics

The Rhino Robot is a 4 degree-of-freedom robot used to illustrate programming and control of robot manipulators for classroom settings.

Rhino Robot:
http://www.theoldrobots.com/images40/rinoarm4.JPG

Reference Frames



Link i	α_{i-1} The angle between the Zi-1 and Zi axis (twist)	a_{i-1} The distance from Zi-1 to Zi measured along the Xi-1 axis	d_i The distance from Xi-1 to Xi measured along the Zi axis (cm)	Q_i The angle between Xi-1 and Xi measured about the Zi axis
1	0 deg	0	d1 = 50	Q1
2	-90 deg	0	0	Q2
3	0 deg	a2 = 50	0	Q3
4	0 deg	a3 = 50	0	Q4
5	-90 deg	0	d5 = 5	Q5

Forward Kinematics:

Change 4 lines of RRR routine:

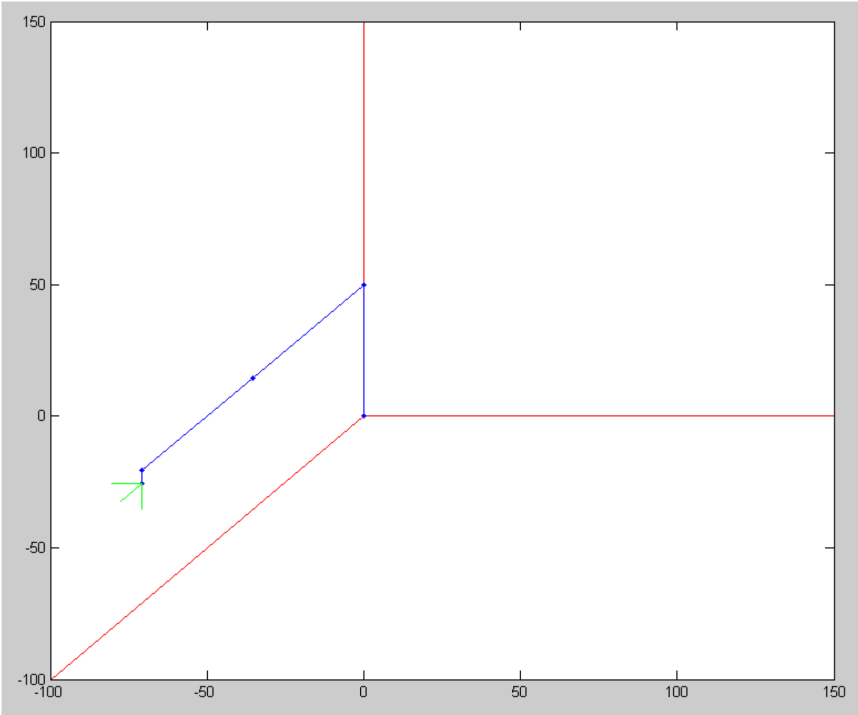
```
Q     = [W(1), W(2), W(3), W(4), W(5)];
alpha = [0, -pi/2, 0, 0, -pi/2];
a = [0, 0, 50, 50, 0];
d = [50, 0, 0, 0, 5];

etc.
```

Calling routine:

```
Rhino([0,0,0,0,0], zeros(4,1))

x  100.0000
y    0.0000
z   45.0000
     1.0000
```



Inverse Kinematics: Algebraic Solution:

Given the joint angles, the tip position is known via transformation matricies.

The net result for a Rhino robot is the tip relative to reference frame 0 is:

$$P_0 = T_{01}T_{12}T_{23}T_{34}P_4$$

or

$$P_0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & 50 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & 50 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

There is some redundancy in the Rhino robot

- Four joint angles
- 3 constraints (x, y, z)

Going to the wrist gives a unique (ish) solution

$$P_0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & 50 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Multiplying this out...

$$P_0 = \begin{bmatrix} ((50c_3 + 50)c_2 - 50s_2s_3)c_1 \\ ((50c_3 + 50)c_2 - 50s_2s_3)s_1 \\ -(50c_3 + 50)s_2 - 50c_2s_3 + 50 \\ 1 \end{bmatrix}$$

giving 3 equations and 3 unknowns:

$$x = ((50c_3 + 50)c_2 - 50s_2s_3)c_1$$

$$y = ((50c_3 + 50)c_2 - 50s_2s_3)s_1$$

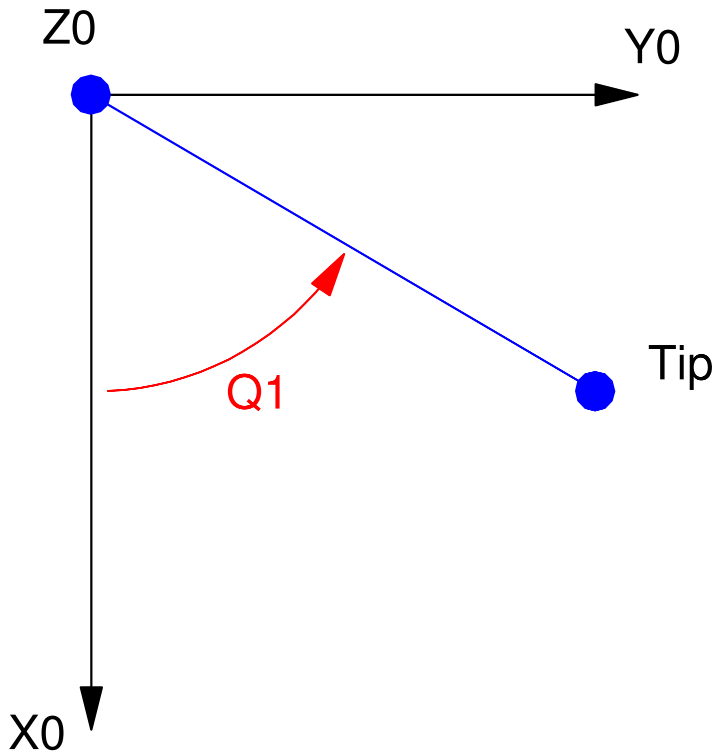
$$z = -(50c_3 + 50)s_2 - 50c_2s_3 + 50$$

These equations are not easy to solve...

Inverse Kinematics: Geometric Solution

The top view of a Rhino Robot tells you Q1:

$$\theta_1 = \arctan\left(\frac{x_{tip}}{y_{tip}}\right)$$



Top view of a Rhino robot

Finding Q2 and Q3:

$$r = \sqrt{x_{tip}^2 + y_{tip}^2}$$

$$d = \sqrt{r^2 + (z-50)^2}$$

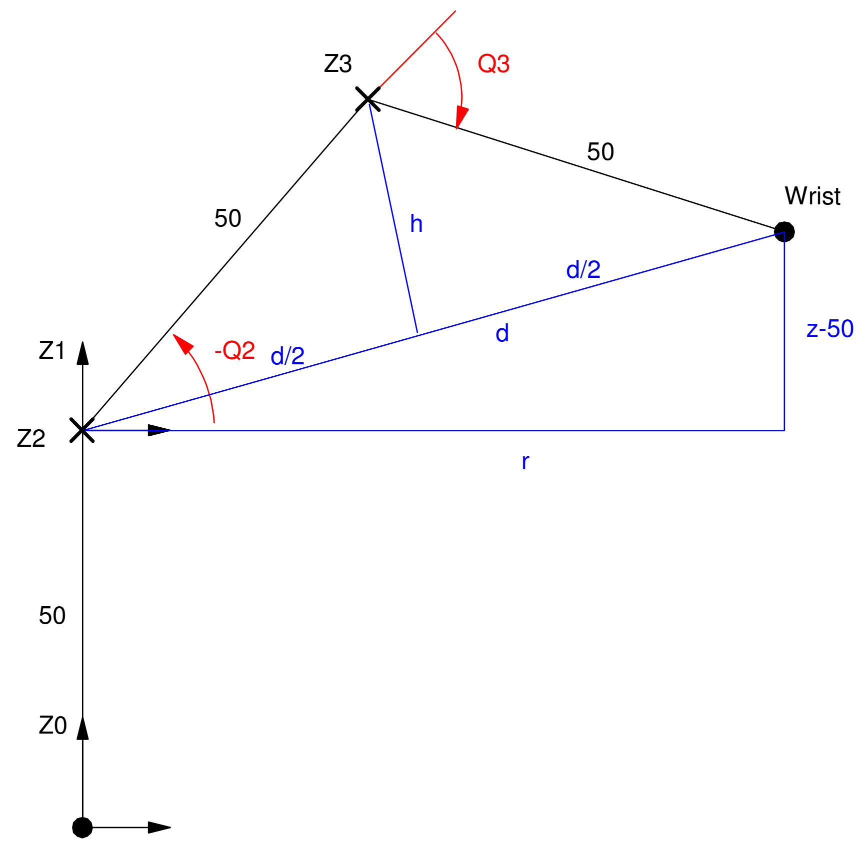
$$h = \sqrt{50^2 - \left(\frac{d}{2}\right)^2}$$

$$\theta_a = \arctan\left(\frac{z-50}{r}\right)$$

$$\theta_b = \arctan\left(\frac{h}{d/2}\right)$$

$$\theta_2 = \theta_a + \theta_b$$

$$-\theta_3 = 180^0 - 2 \cdot \arctan\left(\frac{d/2}{h}\right)$$



This is the program InverseRhino:

- Assumes the wrist is pointing straight down
- $\theta_2 + \theta_3 + \theta_4 = 0$
- *Much* easier solution

```
TIP = Rhino([0.2,0.3,0.4,-0.7,0],zeros(4,1))

   84.2945
   17.0873
   -1.9869
    1.0000


Q = InverseRhino(TIP)

    0.2000
    0.3000
    0.4000
   -0.7000
         0
```