Inverse Kinematics for a Puma Robot

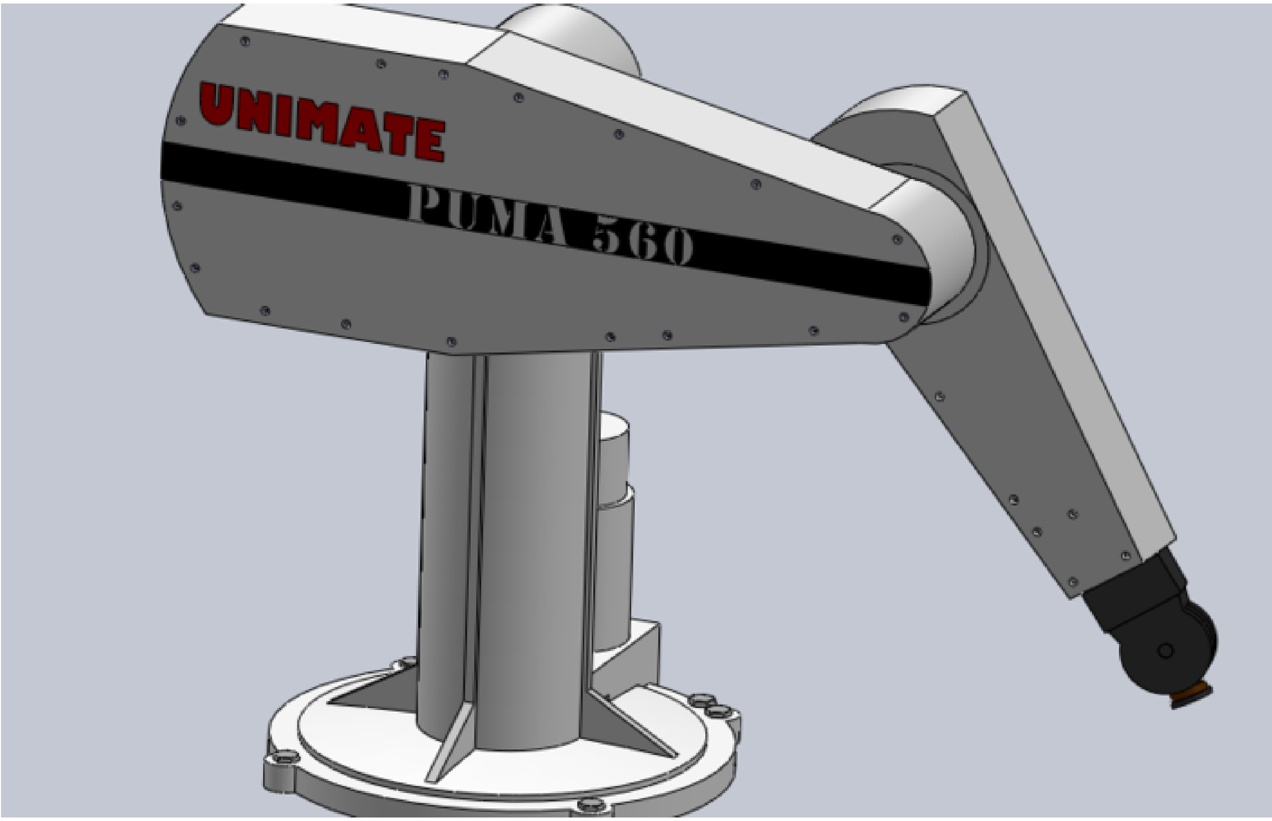
Lecture #6

ECE 761: Robotics

Class taught at North Dakota State University
Department of Electrical and Computer Engineering

Puma Robot

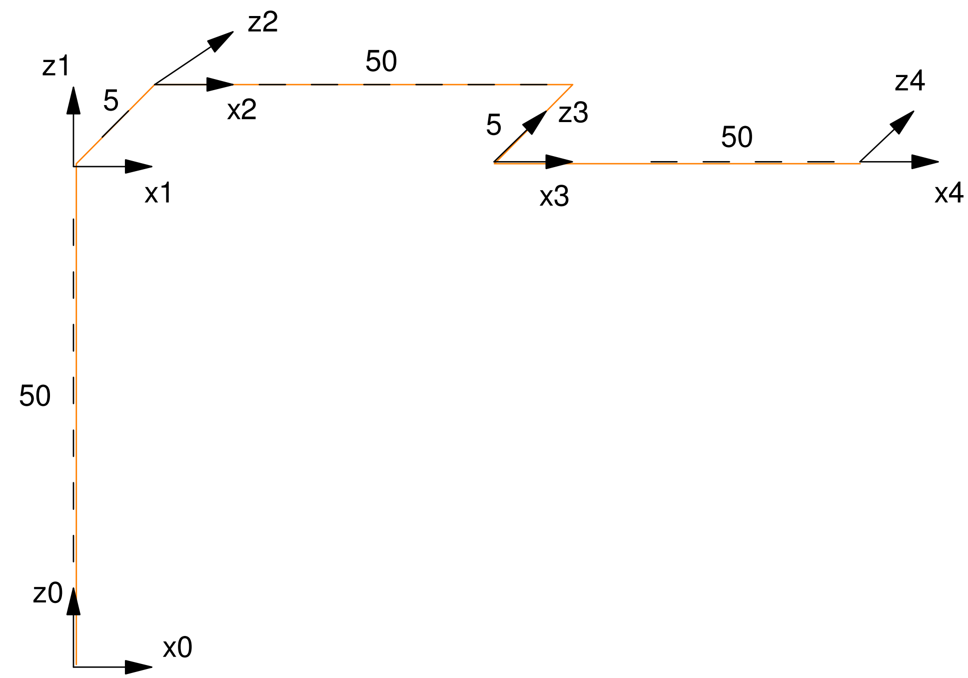
- Programmable Universal Machine for Assembly
- RRR Robot produced by Unimation in 1978
- Still in use



Reference Frames (Forward Kinematics)

Simplified Model

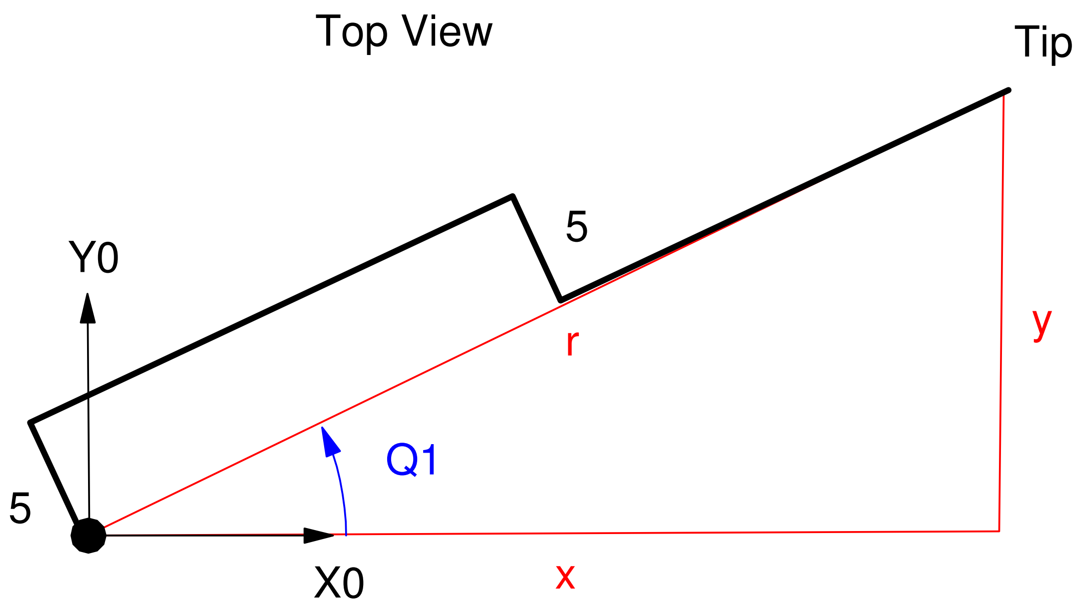
- 50cm & 5cm links
- Offset d3 + d4 = 0
- Similar to Rhino robot



Link i	α_{i-1} The angle between the Zi-1 and Zi axis (twist)	a_{i-1} The distance from Zi-1 to Zi measured along the Xi-1 axis	d_i The distance from Xi-1 to Xi measured along the Zi axis	Q_i The angle between Xi-1 and Xi measured about the Zi axis
1	0	0	50	Q1
2	-90 deg	0	5	Q2
3	0	50	-5	Q3
4 (tip)	0	50	0	0

Inverse Kinematics

- Determine the joint angles given the tip position
- Since d2 + d3 = 0, the net effect is the tip is inline with the base of the robot.
- $\theta_1 = \arctan\left(\frac{y_{tip}}{x_{tip}}\right)$



Top View of the RRR robot. Note that the two offsets cancel resulting in Q1 point to the tip.

Side View:

- Similar to Rhino robot

$$a = z_{tip} - 50$$

$$b = \sqrt{x_{tip}^2 + y_{tip}^2}$$

$$r = \sqrt{a^2 + b^2}$$

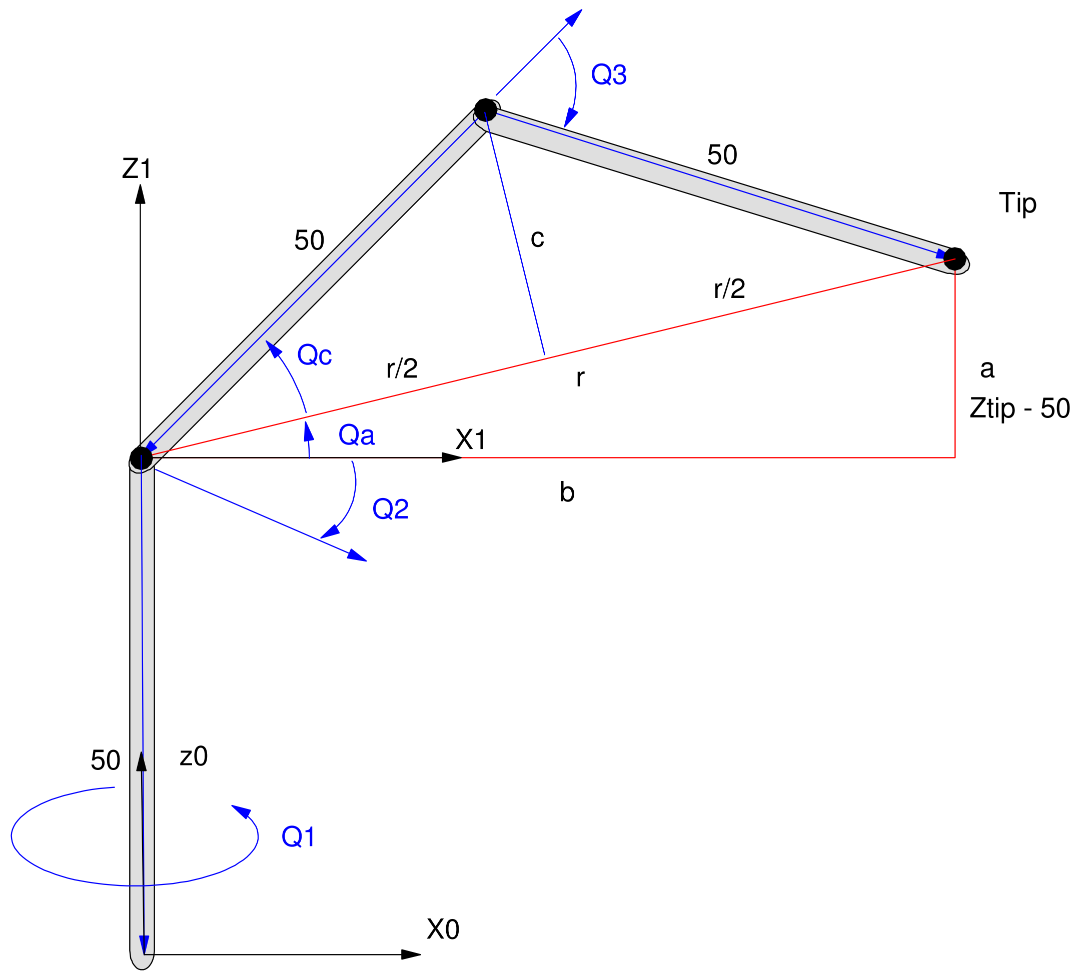
$$c = \sqrt{50^2 - \left(\frac{r}{2}\right)^2}$$

$$\theta_a = \arctan\left(\frac{a}{b}\right)$$

$$\theta_c = \arctan\left(\frac{c}{r/2}\right)$$

$$\theta_2 = -(\theta_a + \theta_c)$$

$$\theta_3 = 2\theta_c$$



Case 2: d2 + d3 ≠ 0

For the Pume robot, d2 and d3 are *not* the same.

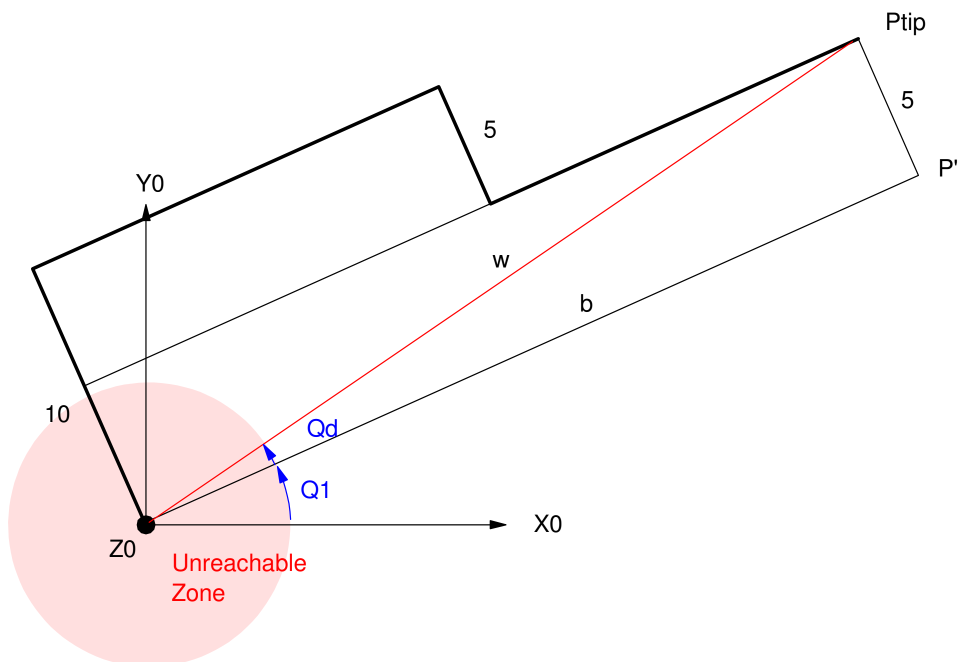
Link i	α_{i-1} The angle between the Zi-1 and Zi axis (twist)	a_{i-1} The distance from Zi-1 to Zi measured along the Xi-1 axis	d_i The distance from Xi-1 to Xi measured along the Zi axis	Q_i The angle between Xi-1 and Xi measured about the Zi axis
1	0	0	50	Q1
2	-90 deg	0	10	Q2
3	0	50	-5	Q3
4 (tip)	0	50	0	0

This isn't a problem for forward kinematics

It is a problem for inverse kinematics

Top View

- There is a cyllinder about the Z0 axis with a radius of 5cm where the robot cannot reach
- The equations for the inverse kinematics get a bit more complicated.



Top View of the PUMA Robot. The shoulder and elbow offsets do not cancel, resulting in a 5cm offset for the tip

First, determine the joint angle, Q1

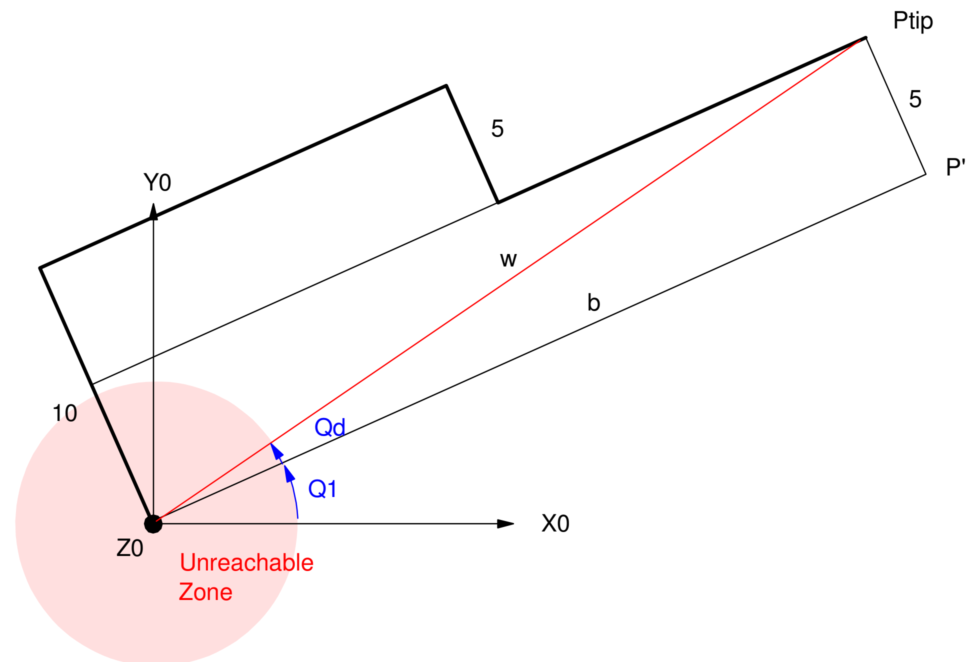
$$w = \sqrt{x_{tip}^2 + y_{tip}^2}$$

$$b = \sqrt{w^2 - 5^2}$$

$$\theta_{tip} = \arctan\left(\frac{y_{tip}}{x_{tip}}\right)$$

$$\theta_d = \arctan\left(\frac{5}{b}\right)$$

$$\theta_1 = \theta_{tip} - \theta_d$$

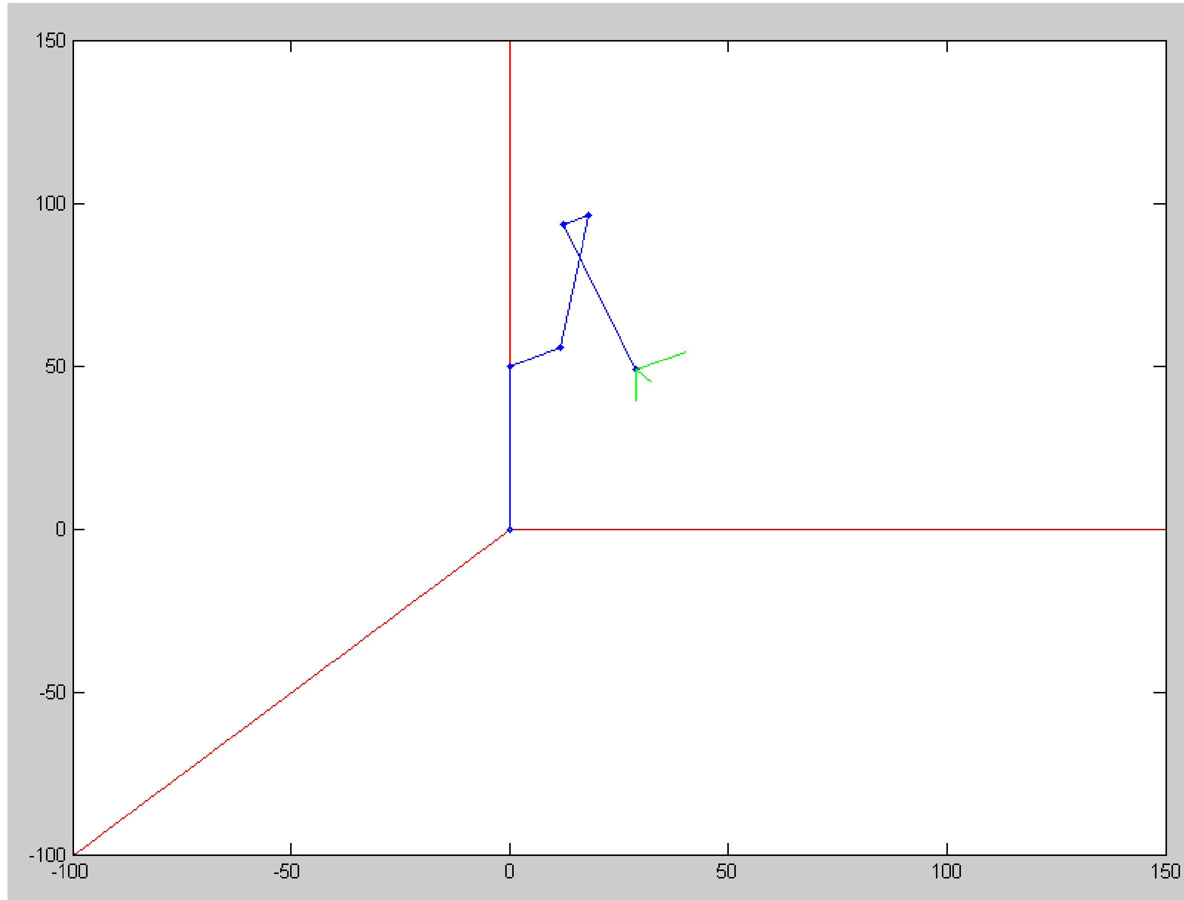


From this point on, the previous equations all apply, with the note that 'b' is the value computed here rather than the distance to the tip (called 'w' here)

Matlab Code

Program InversePuma

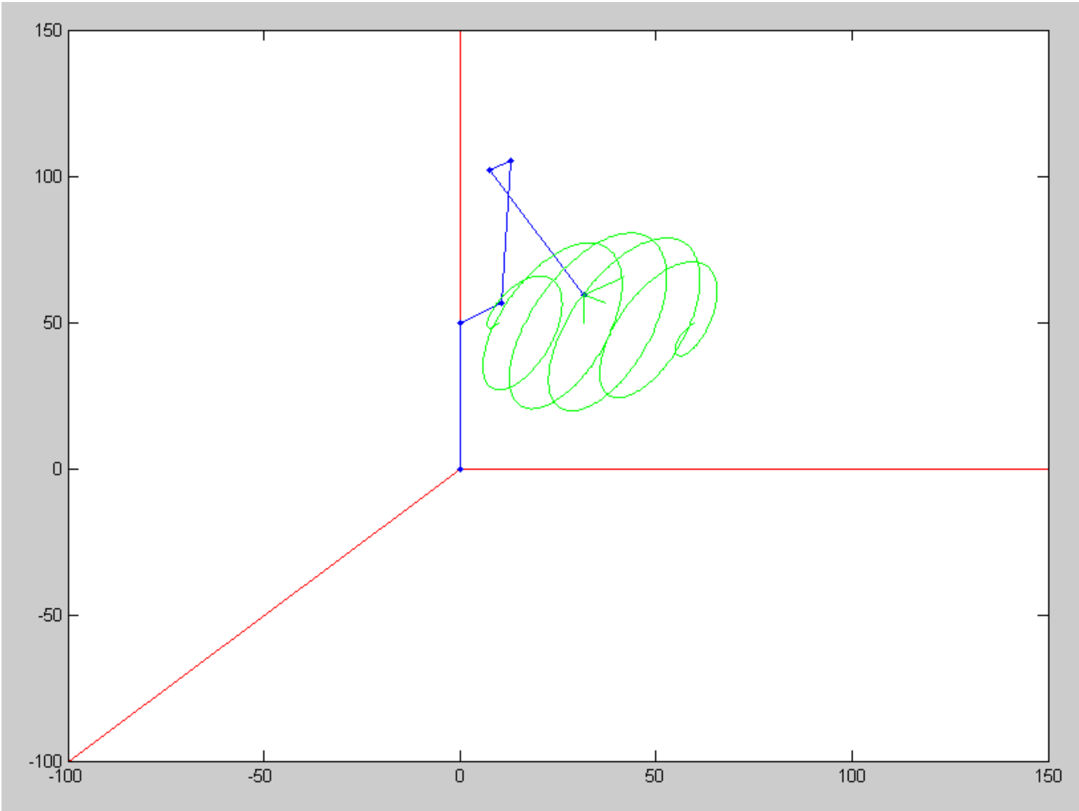
```
TIP = [30,50,70]';
Q = InversePuma(TIP)

     0.9445
    -1.2407
     1.8183
    -0.5776
          0

Puma(Q, zeros(4,1))

     30
     50
     70
      1
```



Using Inverse Kinematics:

With inverse kinematics, you can trace out a shape

```
% units = meters
t = [0:0.001:1];
y = t;
r = (0.25 - (y-0.5).^2) .^ 0.5;
x = r .* cos(10*pi*t);
z = r .* sin(10*pi*t);

% units = cm

TIP = [50*x;50*y+10;50*z+50;x.^0];



npt = length(t);

for i=1:npt

   Q = InversePuma(TIP(:,i));
   Puma(Q, TIP);

   end
```



Homework #6

- Find the forward and inverse kinematics for two robots
- Demonstrate the two are consistent