Position Control of a Robot with Gears

Lecture #12

ECE 761: Robotics

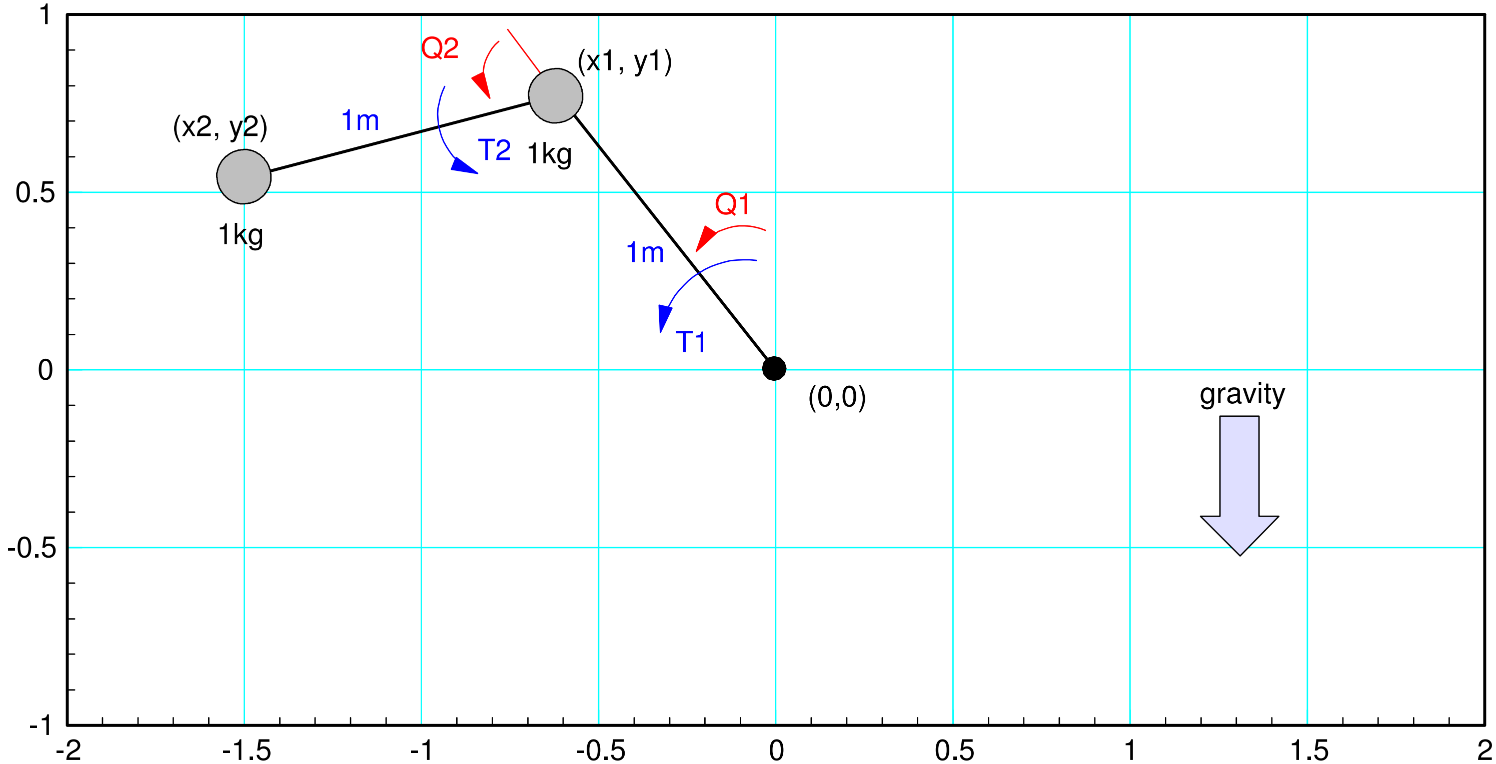
Class taught at North Dakota State University
Department of Electrical and Computer Engineering

Please visit www.BisonAcademy.com for corresponding lecture notes, homework sets, and solutions.

Position Control of a Robot with Gears

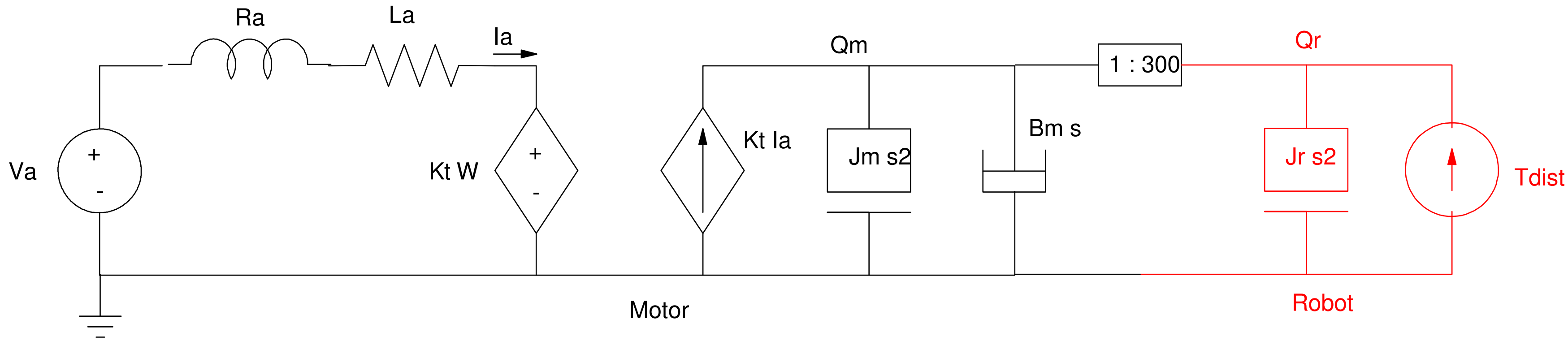
Problem: Control the tip position of a 2-link robotic arm

$$\begin{bmatrix} 2(1+c_2) & (1+c_2) \\ (1+c_2) & 1 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \begin{bmatrix} 2s_2\dot{\theta}_1\dot{\theta}_2 + s_2\dot{\theta}_2^2 \\ -s_2\dot{\theta}_1^2 \end{bmatrix} - g\begin{bmatrix} 2c_1 + c_{12} \\ c_{12} \end{bmatrix}$$



Equations for a Brush-Type DC Motor (DC Servo Motor)

The two are coupled by the torque constant, Kr:



Model for a DC Servo Motor connected to a robot with a 300:1 gear reduction

The dynamics for a DC servo motor are:

$$(J_m s^2 + B_m s)\theta_m = K_t I_a$$

$$V_a = (Ls + R)I_a + K_t s\theta_m$$

which simplifies to

$$\theta_m = \left(\frac{1}{s}\right)\left(\frac{K_t}{(J_m s+B_m)(L_a s+R_a)+K_t^2}\right)V_a$$

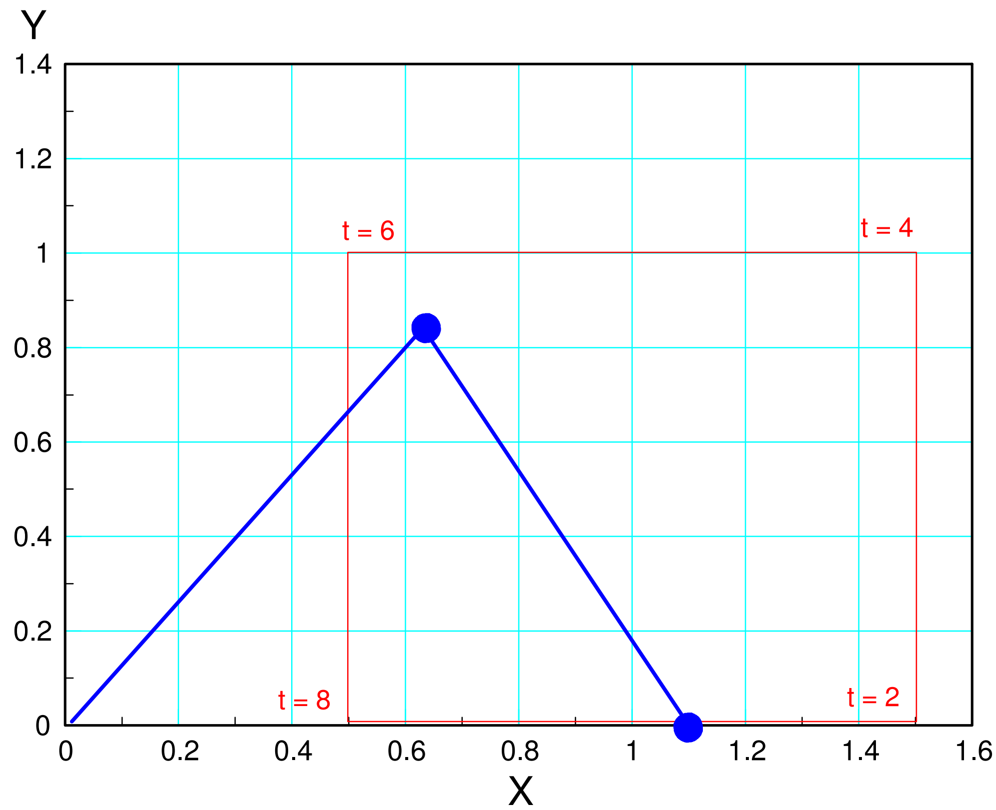
If L = 0 (typical)

$$\theta_m = \left(\frac{1}{s}\right)\left(\frac{K_t}{(J_m s+B_m)(R_a)+K_t^2}\right)V_a$$

Motor Sizing:

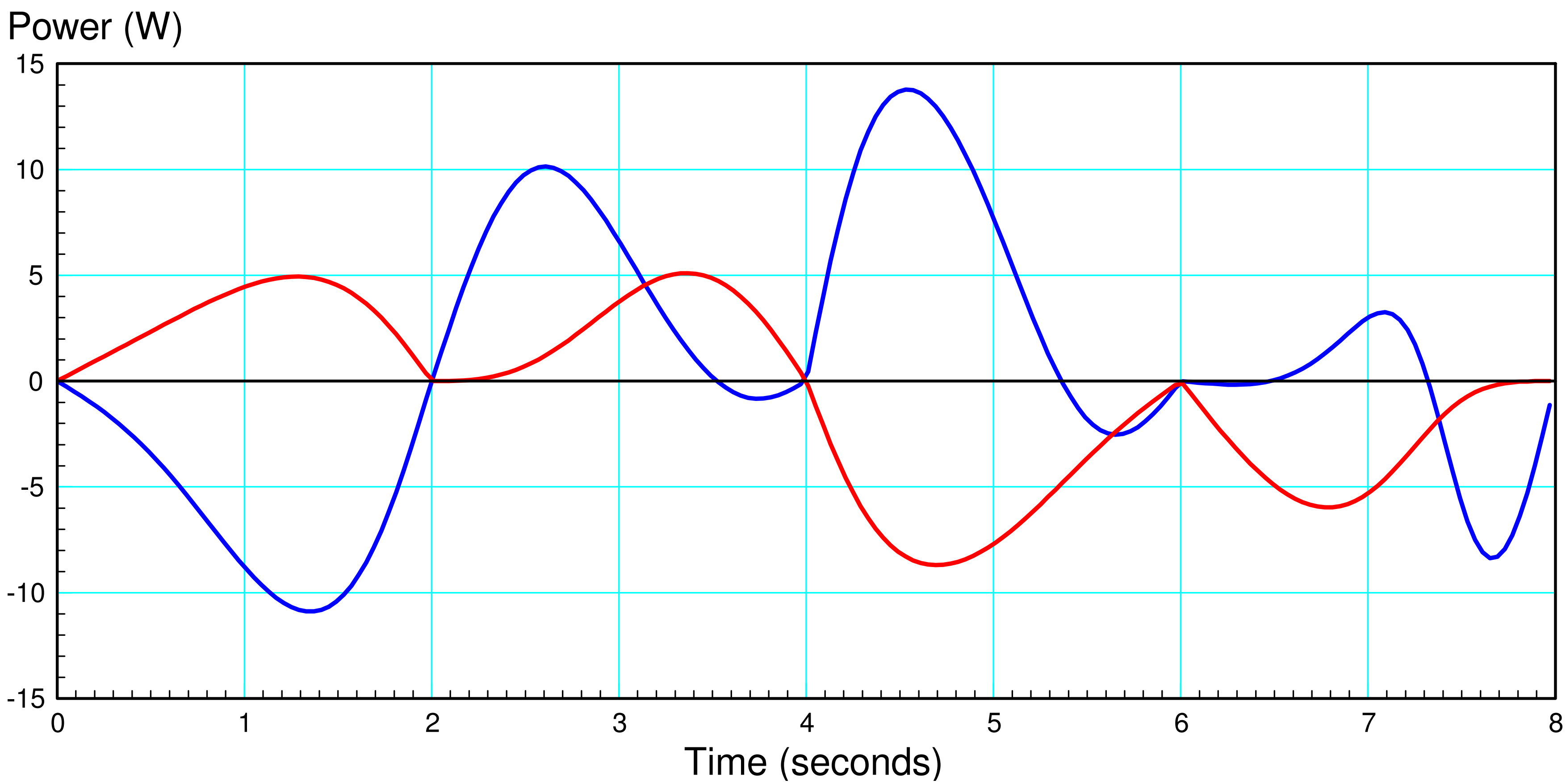
Power = V I = Torque * Speed

- Define the path you want to follow (trace out a square)
- Compute the required torques
- Compute the joint velocities
- Compute the power the motor must deliver



Pittman 14207 Motor operated at 76.4DC (100 Watts) will work

$$\theta = \left(\frac{714}{s(s+612)}\right) V_a$$



Gears:

- Increase the torque of the motor, and
- Reduce the impedance the motor sees

GR-231012H Ratio 12:1
Download GR-231012H.PDF

- Backlash < 10 arc-min
- Gear Ratios 12:1
- Nominal Torque 35 NM
- Front & back frame standard Nema23
- Maximum Input RPM 5000
- Rated Input RPM 3000
- Input clamping : Coupling
- Output shaft diameter 12.7mm/0.5"
- Output shaft length 32mm/1.26"
- Input shaft diameter 6.35mm/1/4", 8mm/5/16", 12.7mm/0.5", 14mm/0.551"
- Input shaft length 36mm/1.41"
- Lifetime Lubrication

Buy it Now $320
Add To Cart

Gear Reducer: http://motiontek.ca/gear_reducer.html

Gears vs. Impedance

Gears reduce the speed of the shaft

$$\omega_1 = \left(\frac{N_2}{N_1}\right)\omega_2$$

Gears increase torque (power has to balance)

$$T_1 = \left(\frac{N_1}{N_2}\right)T_2$$

The impedance seen by a motor drops as the turn ratio squared

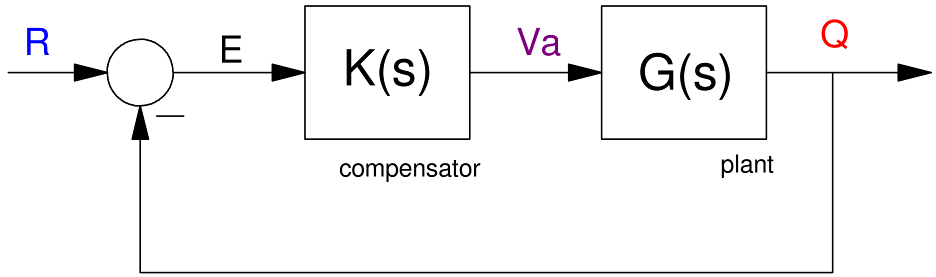
$$Z_1 = \frac{T_1}{\omega_1} = \left(\frac{\frac{N_1}{N_2}T_2}{\frac{N_2}{N_1}\omega_2}\right) = \left(\frac{N_1}{N_2}\right)^2\left(\frac{T_2}{\omega_2}\right)$$

If you gear down the motor, the outside world doesn't matter

- All you need to do is control the position of the motor

Angle Control of a DC Motor

Assume a unity feedback:



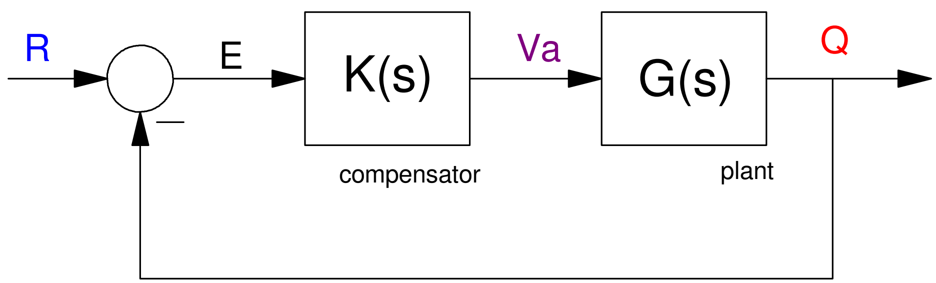
G(s) = Motor + Gear + Robot

- Essentially just the motor and the gear

$$G(s) = \left(\frac{714}{s(s+162)}\right)$$

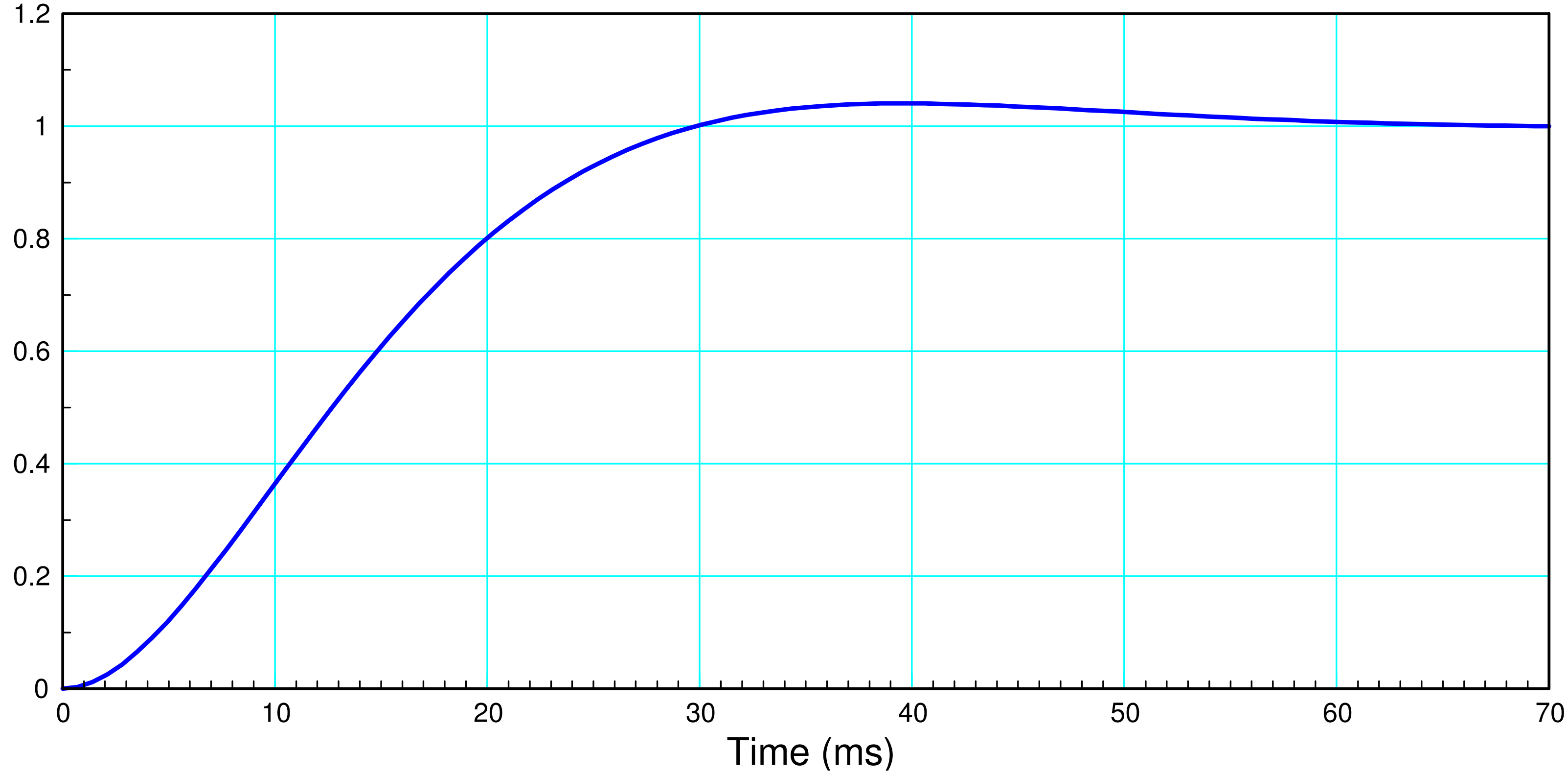
P Control

- Measure motor angle
- Only use desired angle (R)
- $\theta = \left(\frac{714P}{s(s+162)}\right)E$
- $\theta = \left(\frac{714P}{s(s+162)+714P}\right)R$



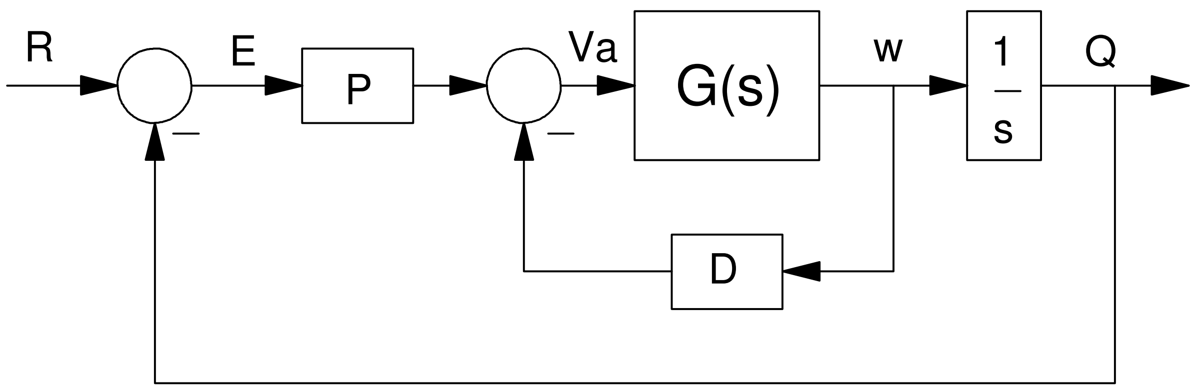
Step Response

- P = 18
- $\theta = \left(\frac{12852}{s(s+162)+714P} \right) R$



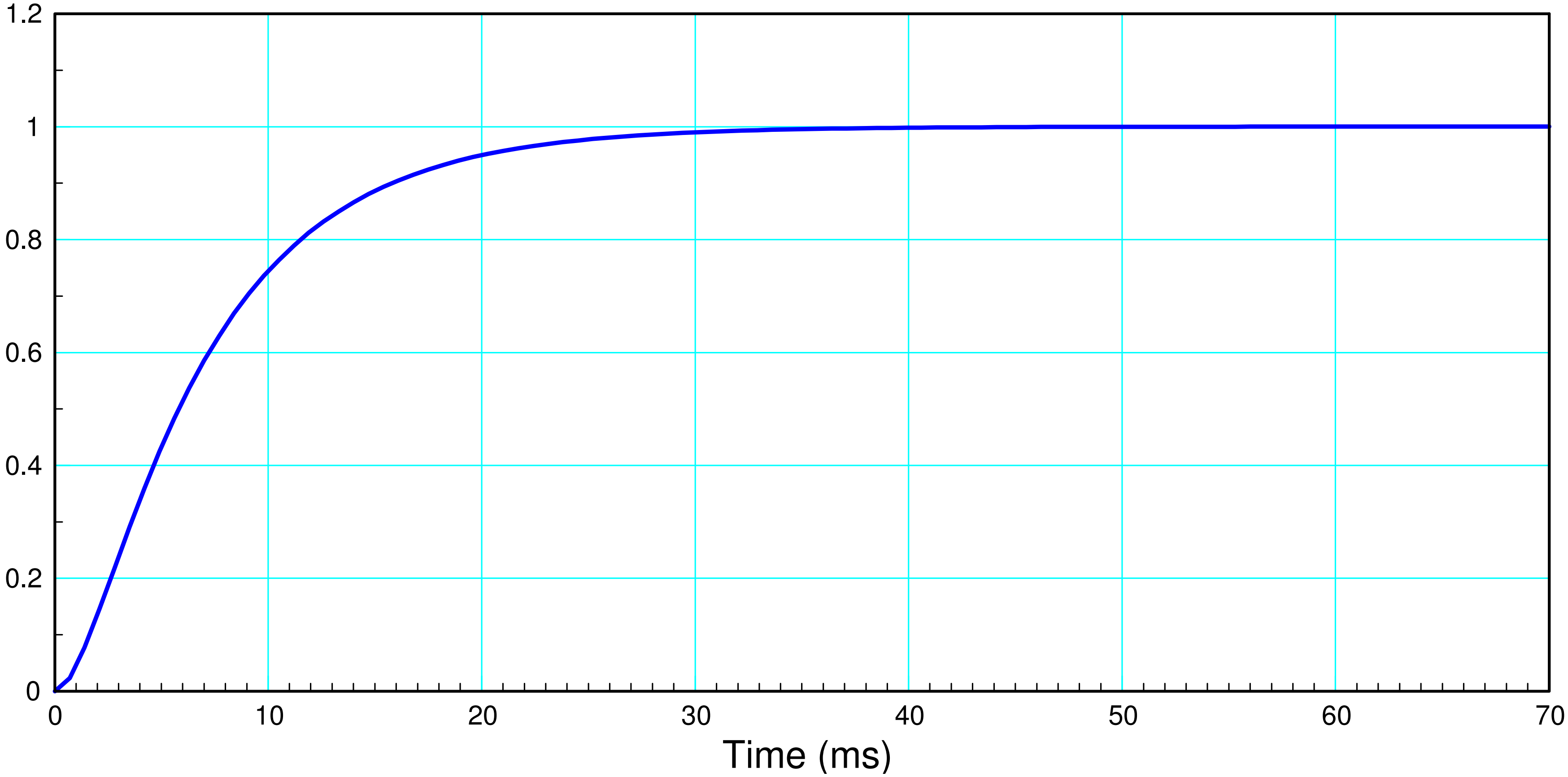
PD Control

- Measure motor speed (D*w)
- Only uses desired angle (R)
- $\theta = \left(\frac{714P}{s(s+162+714D)}\right)E$
- $\theta = \left(\frac{714P}{s^2+(162+714D)s+714P}\right)R$



Step response

- D = 1, P = 162

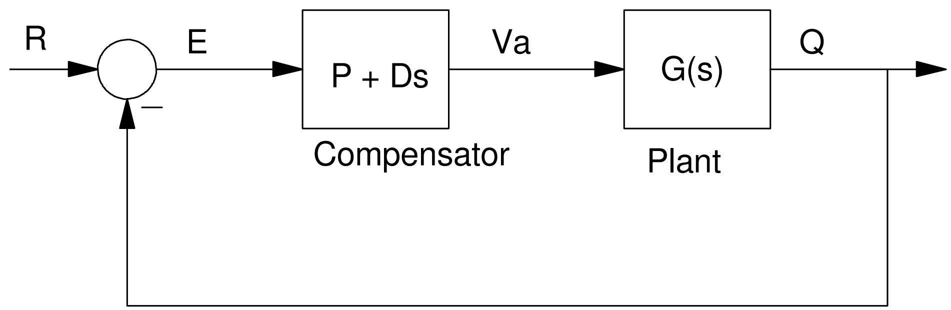


PD Control (take 2)

- Measure angle and velocity
- R contains both desired angle as well as derivative
- $\theta = \left(\frac{714(Ds+P)}{s(s+162)}\right)E$

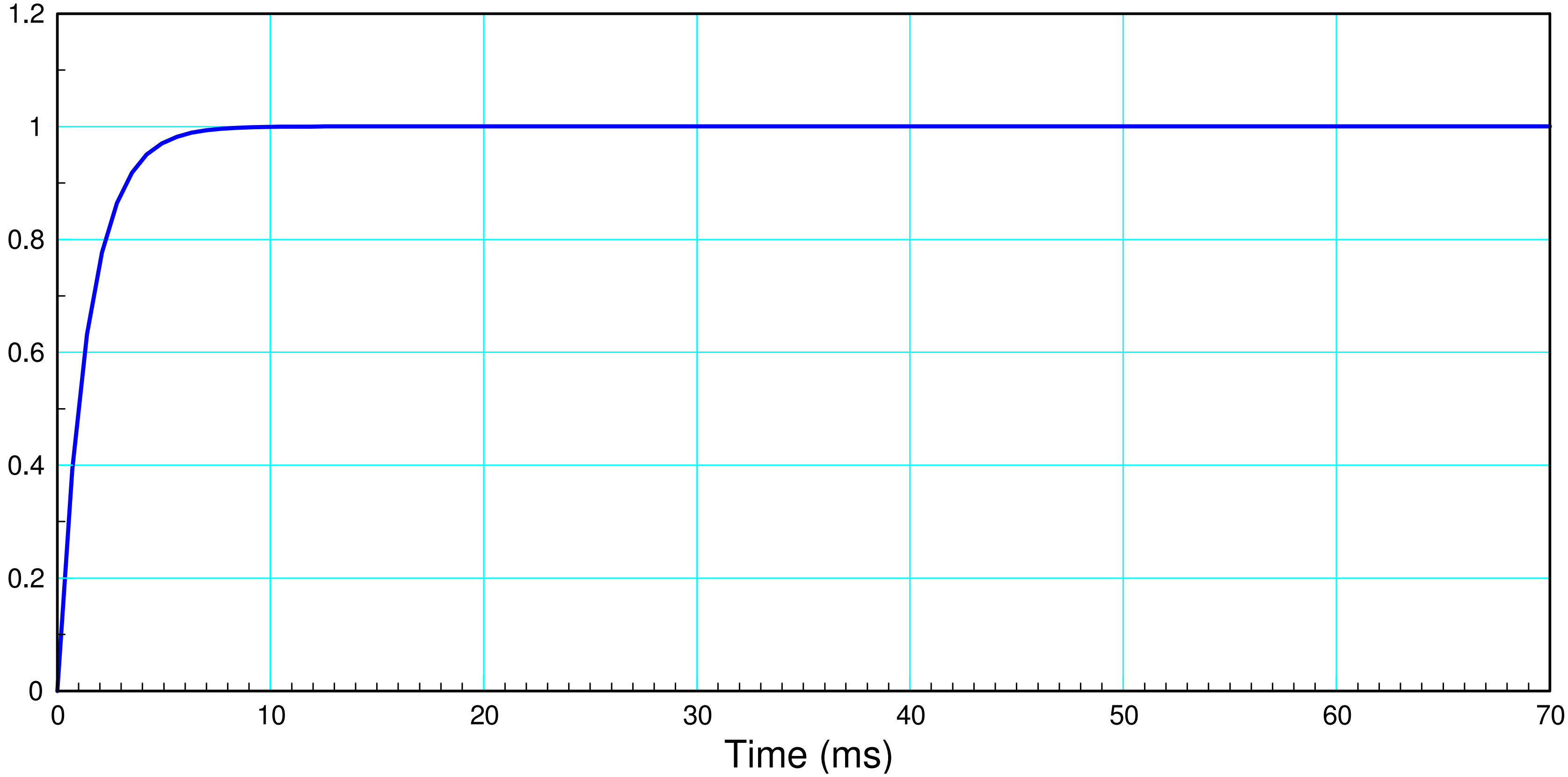
Let Ds + P = k(s+162)

- $\theta = \left(\frac{714k}{s+714k}\right)R$



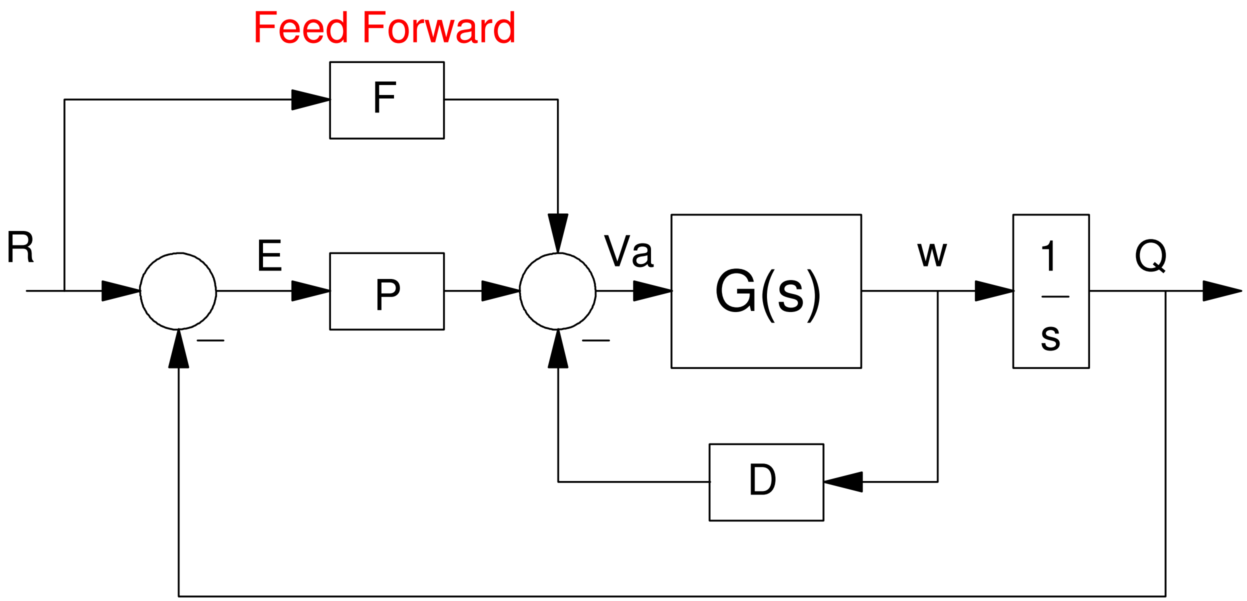
Step Resposne

- D = 1, P = 162



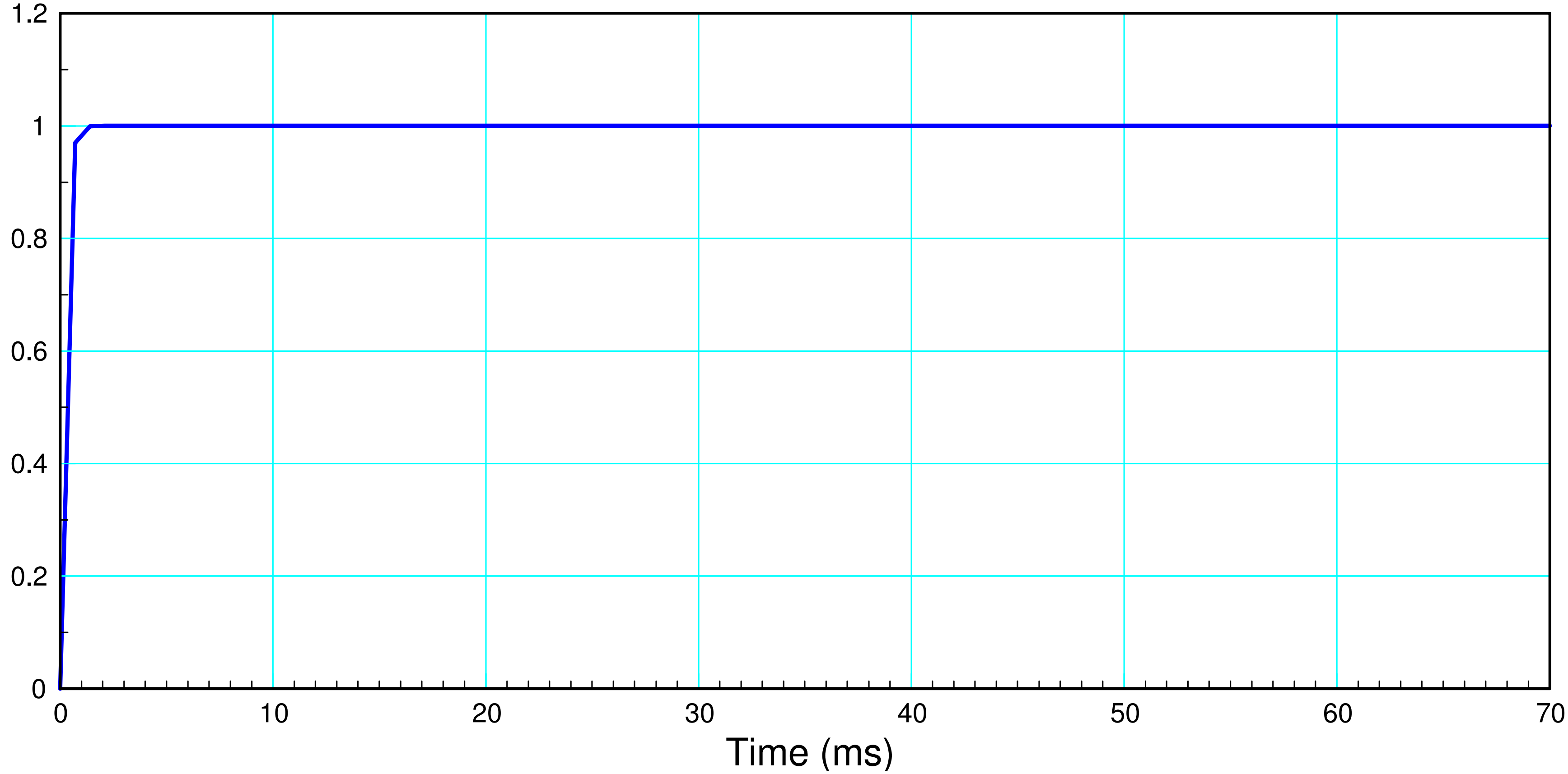
PD Control plus Feed Forward Control

- Measure angle and velocity
- Apply desired angle (R), velocity, and acceleration
- $\theta = \left(\frac{714P}{s(s+162+714D)}\right)E$
- $\theta = \left(\frac{s^2+(162+714D)s+714P}{s^2+(162+714D)s+714P}\right)R = R$



Step Response

- D = 1, P = 162



Simulation:

- Compute the desired angle (R)
- Find the output of the motor

$$\theta = \left(\frac{714k}{s+714k}\right)R$$

- The motor's angle is the actual robot angle